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SVM-9

Equivalence Techniques for Vibration Testing

Warren C. Fackler

Collins Radio Company

1972

The Shock and Vibration Information Center
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June 1971

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CHAPTER 1
INTRODUCTION

The purpose of this monograph is to present a critical review of those equivalence techniques which have been and are being used to define and simulate service vibration environments in the testing laboratory. A further objective of the monograph is to present a unified and current overview of the heretofore fragmented activities involved in the formulation of vibration equivalence methods.

The term *vibration equivalence* shall be interpreted to be concerned with (a) the various techniques used to derive test levels, (b) the performance of tests intended to simulate the conditions of service vibration, and (c) the duplication of critical damage processes. The interpretation is extended to include those techniques which may be applied to derive substitute vibratory loads for use in product design and performance evaluation. As a result of the previous interpretation, the vibration equivalences are found to encompass vibration simulation techniques in general, and are not restricted to the unsuccessful random-sine equivalences which were attempted during the middle 1950's.

The vibration equivalence techniques were assembled in two categories, as shown in Fig. 1-1, using equality as a basis for defining each category. One category includes those theories which establish equality by defining a damage criterion and equating the amount of damage produced by different vibration experiences. This category of equivalence techniques includes damage that will accumulate and eventually cause failure, and damage that is a function of the magnitude of some parameter of specimen response. The second category of equivalence techniques includes those approaches used to improve test realism by recognition of the ever-present structural interaction between a specimen and its support.

The equivalences based on cumulative damage draw heavily on various theories developed to explain material fatigue processes. The fundamental theory needed to develop the cumulative damage equivalences is developed by Chapter 3, and will lead to various techniques which may be used for scaling test time and changing test type. The equivalences based on magnitude of damage include those techniques which equate important specimen response characteristics such as motion or operational malfunction. These equivalences are described in Chapter 4. Chapter 5 covers the interaction equivalences and is to a large extent a survey of current work in the application of mechanical-impedance concepts to the problem of improving test realism. The mechanical-impedance equivalences are difficult to implement because in-service force data are not generally available or easy to obtain. The emphasis in this chapter is on providing the reader

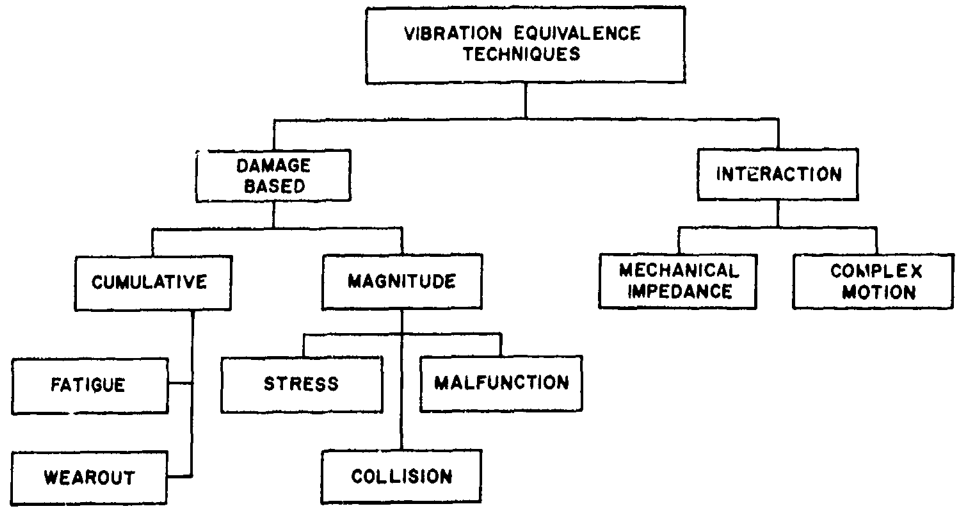

Fig. 1-1. Vibration equivalence categories.

with an appreciation for possible weaknesses in test and equivalence practices when specimen and support interactions are ignored. Chapter 6 provides a discussion of the uses of vibration equivalence techniques and the application of equivalence concepts to the problem of performing meaningful vibration tests. Although the word *test* appears frequently in the text, no lack of emphasis is implied for the problem of field data interpretation. An objective of Chapter 6 is to identify each major event associated with vibration simulation and testing, discuss the relationship between each event and the other events, and indicate the manner in which the equivalence techniques are used to define the interrelationship of these events.

The application of vibration equivalence techniques to accepted testing procedures involves the use of engineering judgment, empiricism, and a willingness to accept some inaccuracy in results. Therefore, the use of vibration equivalence techniques has been subject to some controversy [1], even though the boundaries of this inaccuracy are better defined than the boundaries of the inaccuracies which may result from certain current test practices.

Included in this monograph are the results of a thorough survey and literature search directed toward locating every available paper relating to vibration equivalence. The primary sources of information include the following:

1. All unclassified *Shock and Vibration Bulletins* through 1970.
2. *The Applied Sciences and Technology Index*, 1958 through February 1971.
3. A literature search performed by the Defense Documentation Center, covering 20 years to March 1970.
4. A literature search performed by the National Aeronautics and Space Administration, covering 20 years to March 1970.
5. *The Shock and Vibration Digest*, January 1969 through March 1971.

6. References cited in reviewed articles.

7. Suggested additional sources resulting from a letter survey of 186 authors and engineers in the field of vibration testing.

Each article, paper, or report was read and annotated for inclusion in the monograph. The papers are widely distributed in source, with many from countries outside of the United States. It was found that the majority of the authors presupposed that the reader was familiar with work in the fields of fatigue damage and mechanical impedance. The monograph contains rudimentary theory in both of these fields for the uninitiated reader.

The letter survey of authors and engineers engaged in vibration testing elicited a generous response. A few of the letters are quoted which represent the general thinking of the respondents. Several unpublished memos, papers, and reports were also surveyed; however, the bibliography is limited to available published work.

One persistent problem which tended to hamper the review and comparison of the many authors' work was an inconsistency in the use of symbolism and a corresponding lack of a symbolism technique which would define accurately the meaning of important symbols. A result of this problem was the adoption of a generalized method of symbolism, described in the appendix, which is used throughout the monograph.

The overview of various vibration equivalence techniques and their application, as provided by this monograph, was intended to be complete and current. Therefore, if any significant paper or report was omitted, comments on such omissions will be gratefully received.

CHAPTER 2
A REVIEW OF CUMULATIVE-DAMAGE THEORY

It is important for the reader to be familiar with those fundamental aspects of fatigue theory which provide a basis for the cumulative-damage equivalences. Of particular interest are the concepts associated with the fatigue and cumulative-damage processes. The ideas are not difficult to master; however, it is easy to assume that a material characteristic is of a simple nature when the characteristic is in fact strongly dependent on some less obvious variable. As an example, there are many handbooks, technical articles, and vendor specifications in which the ultimate strength of a material may be found in a listing of material mechanical properties. For the majority of engineering uses the listed value is an adequate guide. However, the engineer who is designing a structure that experiences rapidly changing loads, perhaps aircraft landing gear, might reject a material which meets all other criteria if he did not know that the ultimate strength of certain materials will appear to increase at high rates of strain. Many additional examples could be cited, such as the temperature dependence of ductility and creep, but it is necessary to direct our attention to those material characteristics which influence the use of vibration equivalence techniques.

The first section of this chapter provides a review of fatigue theory and the concept of cumulative damage. This section is intended to serve as a primer and may be bypassed by the reader who is versed in fatigue terminology.

The second section presents detailed coverage of three cumulative damage theories. These theories are of interest because each of the currently used cumulative-damage equivalences may be reduced to a mathematical statement identical to one or more of the three theories. The more advanced reader may choose to bypass this section.

The final section of the chapter contains a listing, by type, of all fatigue theories which were found by the author during a literature search associated with the preparation of this monograph. Each fatigue theory was categorized as either linear, nonlinear, or phenomenological. In addition, each theory is briefly described and suitable reference sources are identified. This section is intended as a starting point for the reader who has interest in the development of more complex equivalence techniques.

There are many excellent publications which may be consulted by the reader who is interested in a more detailed presentation of progress in the field of fatigue damage. These publications include Refs. 2 through 5.

2.1 Fatigue and Cumulative Damage

Fatigue is a process of damage accumulation resulting from the repeated application of load. The term *load* is used to specify any force or collection of forces acting on an object, and is described by known or measured relationships between magnitude, direction, and time. The results of load application are deformation and the formation of stress patterns in the loaded object. It follows that discussions of material fatigue properties in terms of stresses and stress cycles imply specific load patterns on a given object. Thus the term *load* is used to define completely the stress and deformation resulting from forces–whether these forces originate from sources internal or external to an object, result from changes in momentum, are due to thermal expansion, or are caused by forces of any other origin.

A load may consist of one or more applications of a single load, a sequential application of several specific loads, the superposition of one or more single or sequential loads, the superposition of a randomly varying load and single or sequential loads, or perhaps an entirely random variation of load. Each of these load situations influences the fatigue life of a material. The nature of this influence has interested several researchers and caused them to attempt to find a fatigue model which will accommodate the various types of loads. Current fatigue theory is based on the concept of cumulative damage. The concept of cumulative damage is that every load cycle causes incremental damage, which is accumulated until a certain level of damage is reached at which the specimen will fail.

Historians in the field of fatigue damage have noted that fatigue failures caused by the repeated loading of a structure were not given serious consideration prior to the early 1800's [4]. Even then many engineers were reluctant to accept the idea that small repeated loads, including loads above the endurance limit as it is now defined, would cause damage to accumulate and lead to failure. Fatigue failures appear frequently in service and are known to account for the majority of all mechanical fractures [6].

Two phases believed to be involved in the fatigue process are crack initiation and crack propagation [7]. The various theories used to predict fatigue life differ in the treatment given each of these two phases. In later sections of this chapter the differences among the various theories will be discussed; however, it is of interest here to recognize that some theories hold for both phases of damage accumulation, whereas other theories offer separate models for each phase.

The relationship between the stress induced by an applied load and the number of load repetitions which, under a given set of conditions, will cause failure, may be described by a curve similar to that shown in Fig. 2-1. This curve represents the mean of experimental data and is usually plotted to logarithmic scales. Within certain limits the logarithmic curve may be approximated by a straight-line relationship such as curves A or B in Fig. 2-2. These curves are commonly called *S-N* curves, where *S* refers to stress amplitude and *N* to the number of constant-amplitude load applications expected to cause failure.

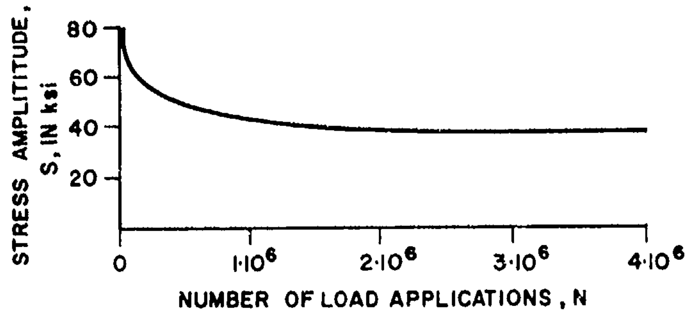

Fig. 2-1. Stress level vs number of load applications.

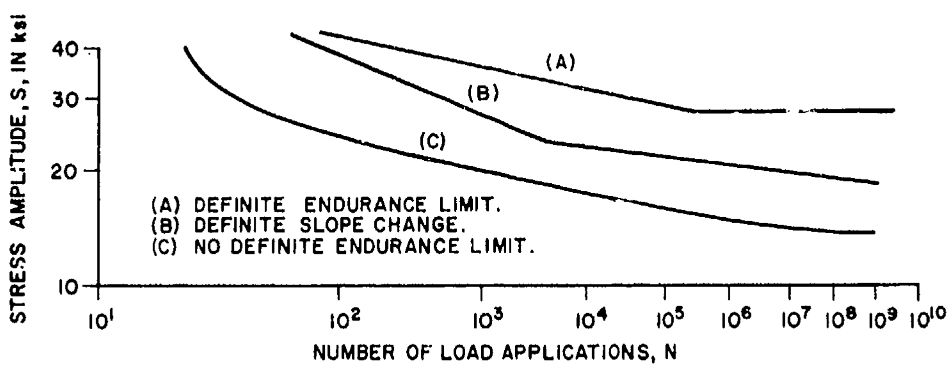

Fig. 2-2. Typical S-N curves.

The number of stress applications that a material will withstand varies with load level and is called *fatigue life*. If a material exhibits the property that repeated stresses below some certain level will not cause failure, that particular stress level is called the endurance limit. Curve A of Fig. 2-2 represents a material which clearly has an endurance limit, whereas an endurance limit is not defined for the materials of curve B or curve C. In those cases in which an endurance limit is not clearly defined, the value of stress at $N = 10^7$ cycles is frequently selected as the endurance limit.

The experimental techniques used to generate S-N curves are usually very carefully controlled to assure uniformity in such factors as specimen geometry, loading, temperature, and alloy composition, and yet at any one load level the test data scatter may range from 10:1 to 100:1. As a result, it is of more significance to describe the S-N curve in terms of a mean value with an attendant standard deviation, or in terms of probability of failure at any specific stress level [8].

When it is necessary to select an S-N curve, consideration must be given to several other factors in addition to assuring that the data were taken for the material of interest and that the range of data scatter is known.

A change in temperature will modify the fatigue failure processes. A reduction in temperature usually increases the number of load cycles needed to cause crack nucleation. At higher temperatures, when a crack is started it tends to propagate faster, and the material tends to fail at shorter crack lengths. An increase in temperature can lead to increased crack propagation rates, as creep processes enhance the progress of fatigue damage. The effect of temperature on fatigue life is shown in Fig. 2-3.

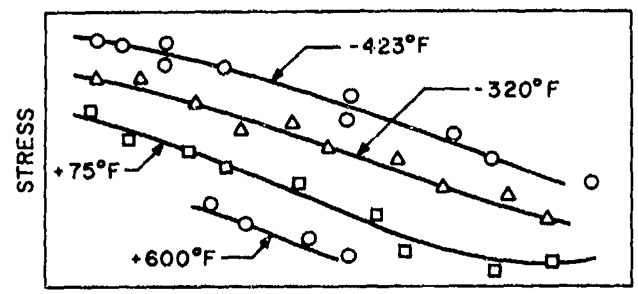

Fig. 2-3. Fatigue life as a function of test temperature (typical for most metals) [9].

The geometrical shape of a specimen, and in particular sharp inside corners, holes, notches, or inclusions that create stress concentration will modify the apparent fatigue properties of the specimen. A stress concentration may create a volume of material subjected to repeated plastic strain. The region subjected to repeated plastic strain may become a source of fatigue cracks which, on continued load application, propagate through the material causing early fatigue failure. The effect of a notch on specimen fatigue life is displayed in Fig. 2-4. Stress risers must be considered during the selection of S-N data for specimens which have notches or other stress risers [6,10].

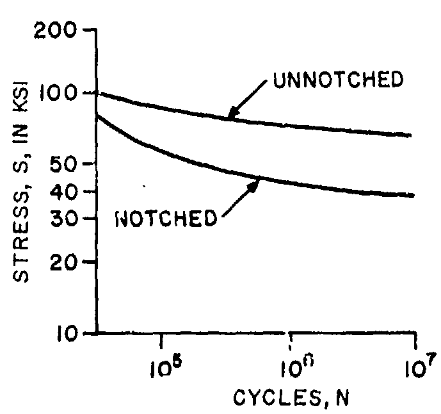

Fig. 2-4. Typical fatigue life reduction due to a notch in a rotating-beam specimen.

The nature of an applied load will influence the experimental S-N relationships. Most fatigue experiments consist of loading a specimen cyclically with a reversing load about a zero mean stress. In Figure 2-5, a zero mean stress condition exists when $\Delta S = 0$ and $S(\max) = -S(\min)$. If some value of mean stress, ΔS, is produced by preloading or the presence of residual stresses, a fluctuating stress is said to exist where $S = \Delta S + S(\text{variable})$. If the absolute value of the mean stress is relatively small, the mean stress has small effect on fatigue life. A "small" mean stress has been estimated to be about one-third of the alternating stress amplitude [8]. When the mean stress is equal to or greater than the alternating stress,

$$|\Delta S| \geqslant |S(\text{variable})| \ , \tag{2-1}$$

the stress is called pulsating and is sometimes characterized by the ratio of $S(\max)$ to $S(\min)$.

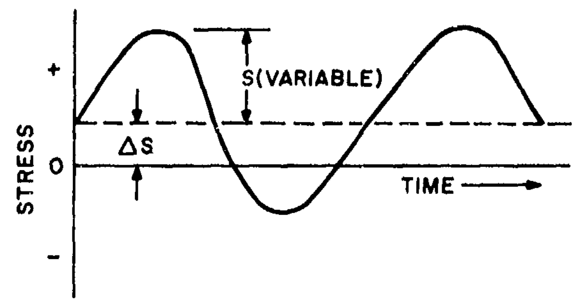

Fig. 2-5. Stress resulting from a cyclic load.

A compressive mean stress or a compressive residual stress must be overcome by an applied load prior to the development of tensile stresses. Compressive mean stresses are frequently used as an intentional design ploy to increase the fatigue life of a critical structure or component. The result is to lower the tensile stresses resulting from a given load, and thus to reduce the tendency to repeatedly open existing fatigue cracks.

The S-N curve is frequently approximated by a straight line on a log-log plot for much of its useful range. This approach is valid in a practical sense because (a) very low-cycle failures approach static strength conditions when compensated for rate of load application, and (b) the high-cycle range has poor accuracy and is usually avoided because small variations in stress lead to very large changes in N. A reason to use caution in the high-cycle range is that damage will accumulate at stresses below the S-N curve [11].

An accurate evaluation of cumulative damage in terms of basic fatigue performance is sometimes questionable, even for a simple test, because the basic mechanics of fatigue damage are poorly understood. Freudenthal [12] tentatively divided the effects of cyclic stress amplitudes into three groups:

1. A high-stress range where $N < 10^5$ cycles. The failures in this range are characterized by severe crystal fragmentation and disorientation accompanied by hardening.

2. A "true" fatigue stress range where $10^5 < N < 10^7$ cycles. The failures in this range are characterized by reversed slip and slip concentration into striations with very little hardening.

3. A "safe" stress range where $N > 10^7$ cycles. In this range there is widely distributed slip, but neither hardening nor substantial pore or microcrack formation occurs.

The simplified S-N curve is illustrated by Fig. 2-6. When the slope of the logarithmic S-N curve is defined as shown, the S-N relationship becomes

$$b \log S = \log C - \log N \ , \tag{2-2}$$

where C is a constant evaluated at a known reference condition. Equation (2-2) is frequently stated in another form,

$$NS^b = C_1 \ , \tag{2-3}$$

where the exponent b assumes values from about 5 to 20 for various materials. When it is of interest to consider changes in N with changes in S, then

$$N_i = N_0 \left(\frac{S_0}{S_i} \right)^b , \tag{2-4}$$

where the subscript 0 refers to a known point on the S-N curve and the subscript i represents the conditions related to or resulting from load i.

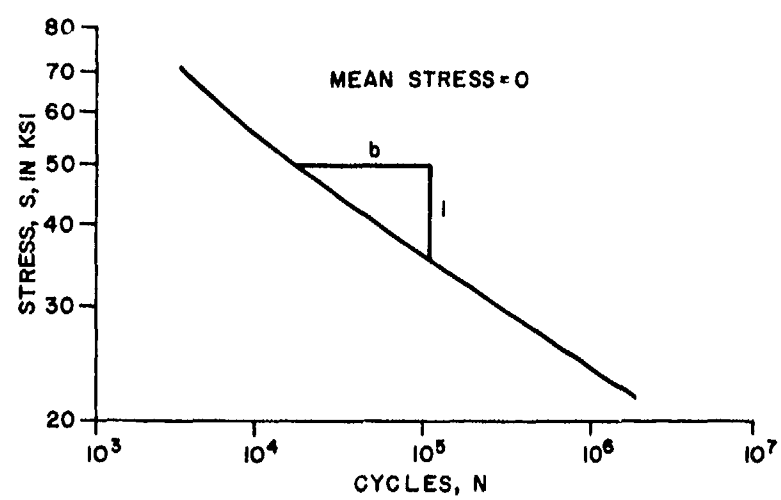

Fig. 2-6. S-N plot for 7075 aluminum.

2.2 Theories Underlying Equivalence

Miner's Method

Miner's [13] method is the most universally applied linear cumulative damage theory because it is relatively simple and yields predictions usually as accurate as other methods. Miner's theory is sometimes called the Palmgren-Miner theory in recognition of the fact that Palmgren [14] documented the linear damage accumulation concept about 20 years before Miner. Langer [7] also investigated linear damage accumulation in a general sense and described the fatigue process as consisting of both crack initiation and crack propagation.

Miner's theory consists of a simple summation of the fraction of usable specimen life consumed at each load level during a specimen's load history. Fatigue damage is assumed to be proportional to work absorbed in the test specimen. The absorbed work in turn is considered proportional to a ratio of the number of applied stress cycles to the number of stress cycles that will produce failure at the given stress level. It is assumed that the amount of damage required to fail a specimen is constant, that the amount of damage is a simple function of load, and that damage is independent of load sequence. Failure is predicted when a sum of the fractional damage from all sources of cyclic stress is equal to unity:

$$D = \sum_{i=1}^{m} \frac{n_i}{N_i} , \tag{2-5}$$

where

d = damage fraction or fraction of consumed fatigue life
n_i = the number of cycles experienced by the specimen at load i
N_i = the number of cycles to failure at load i from an appropriate material S-N curve.

Miner's experimental data for total damage accumulation at actual failure gave D values ranging from 0.61 to 1.45. The variations in damage summation have been verified by other researchers [15,16]. The absolute value of damage at failure appears to be a function of load level and load sequence, with variable-amplitude loads causing significant deviations from the $D = 1$ failure criterion. Freudenthal and Heller [17] have shown that D varies between 0.1 and 1.0 when intermixed stresses are used on smooth unnotched specimens of 2024 aluminum and 4320 steel, and Hillberry [18] found that the Miner summation yielded values from 1.5 to 5.0 for 2024 aluminum under random loading. The range of 0.3 to 3.0 seems to contain all well-mixed loading spectrums; however, coaxing tests have yielded values in excess of 10. The values obtained by monotonically increasing load levels, i.e., coaxing, are not necessarily a realistic representation of a service environment. The use of fatigue theory to establish vibration equivalence relationships is usually concerned with the equality of damage between tests rather than the magnitude of the damage summation. If Miner's theory is used to estimate fatigue life, $D = 1$ is a good average value, and values down to $D = 0.3$ may be assumed if a conservative life prediction is desired.

Miner's cumulative damage theory is considered independent of stress because equal amounts of damage are assumed for equal fractions of life regardless of stress amplitude. It is instructive to plot normalized damage against cycle ratio as in Fig. 2-7. Because there is no stress dependence, one curve such as Fig. 2-7 may be used to characterize the damage accumulation process. It is assumed that the amount of damage caused by any one cycle is dependent only on the number of cycles to failure (life) at the applied load amplitude for that cycle, and not on the total life consumed prior to application of that cycle or the magnitude of previous loading. That is,

$$\Delta D_i = \frac{dD}{d\left(\frac{n}{N}\right)} \frac{1}{N_i}, \tag{2-6}$$

which would not be true if the damage curve changed slope at a specified load level due to the application of prior loads [19].

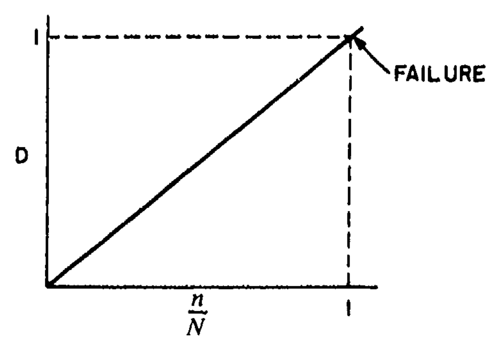

Fig. 2-7. Damage-cycle ratio relationship, no stress dependence.

The definition of damage at failure requires further consideration because a failure mode may be level dependent. A fatigue crack of sufficient depth to cause catastrophic failure at a large load may not cause failure at a lower load. The definition of failure, perhaps crack depth or percent absolute loss of strength, must be uniformly applied to assure the validity of a relationship such as that shown in Fig. 2-7.

The specimen fatigue life N_f in cycles, resulting from j different sets of load history, may be derived from Miner's theory,

$$N_f = \sum_{j=1}^{n} \left[\frac{\sum_{i=1}^{m} n_{ij}}{\sum_{i=1}^{m} \left(\frac{n_i}{n_{ij}} \right)} \right] . \qquad (2\text{-}7)$$

It is now possible to derive an expression for an equivalent stress which will cause the same damage over the total number of applied load cycles as the damage produced by the i stress levels,

$$S_e = \left(\frac{\sum n_i S_i^b}{\sum n_i} \right)^{1/b} , \qquad (2\text{-}8)$$

where S_e is the equivalent stress.

The Miner hypothesis is usually applied to the linear summation of a spectrum of constant-amplitude loads; however, it may be extended to continuous spectrum or random loads. This is accomplished by assuming that each peak in the load spectrum represents a "cycle" and that a characteristic frequency of the narrowband random-load spectrum exists which is defined as the average number of zero crossings with positive slope per unit time. Knowing the probability density of the peaks, the characteristic frequency may be computed,

$$f_c = \frac{E[n_p]}{tP[S]dS} , \qquad (2\text{-}9)$$

where

$E[n_p]$ = the expected number of peaks during time t

f_c = the characteristic frequency of peaks between S and $S + dS$

$P[S]dS$ = the expected number of cycles where the stress amplitudes lie between S and $S + dS$.

In terms of load i, a single-peak stress S_i resulting from load i causes an increment of damage of $1/N_i$. Since the spectrum level varies continuously, the damage summation is expressed by an integral,

$$E[D] = f_c t \int_0^\infty \left(\frac{P[S]}{N[S]} \right) dS , \qquad (2\text{-}10)$$

where

$N[S]$ = the number of cycles to failure at stress amplitude S

$P[S]$ = the probability density of stress peaks

$E[D]$ = the expected value of the damage accumulated over time t.

Equation (2-10) may be evaluated in closed form if the following conditions are satisfied: (a) the load frequency bandwidth is sufficiently narrow to assure that f_c may be evaluated, (b) the stress history is a stationary and normal (Gaussian) process, (c) the fatigue curve can be represented by a straight line in log-log coordinates, and (d) the probability distribution of the stress peaks $P[S]$ is known. When these conditions are satisfied,

$$E[D] = \frac{f_c t}{C_1} (\sqrt{2}\, S)^b \Gamma\left(1 + \frac{b}{2}\right), \tag{2-11}$$

where Γ represents a gamma function and C_1 was defined by Eq. (2-3).

Because it is theoretically possible that peak stress amplitudes will approach infinity, the rms value of stress is frequently used to form a random logarithmic S-N curve. The random logarithmic S-N curve, or logarithmic $\bar{S}$-N curve, can be defined by evaluating Eq. (2-11) at failure,

$$N\bar{S}^b = C_1 \left[2^{b/2} \Gamma\left(1 + \frac{b}{2}\right)\right]^{-1}, \tag{2-12}$$

where $\bar{S}$ is the rms stress, $f_c t = N$, and damage is unity. Root [20] applied Eqs. (2-12) and (2-3) to develop a plot of the ratio of rms random stress to the peak sine stress vs the parameter b, as in Fig. 2-8. Figure 2-9 provides a typical comparison between logarithmic S-N and logarithmic $\bar{S}$-N plots.

The Corten-Dolan Theory

Corten and Dolan [22,23] developed a nonlinear theory for the evaluation of fatigue damage. The Corten-Dolan approach is based on the number of available damage nuclei and rate of crack propagation. The magnitude of the greatest varying load is considered to determine both the number of nuclei and rate of crack propagation. Equation (2-13) expresses the damage accumulation for the i load level,

$$D_i = m_i r_i n_i^{a_i}, \tag{2-13}$$

where

m = number of damage nuclei

r = rate of crack propagation constant

a = an experimentally determined constant exponent.

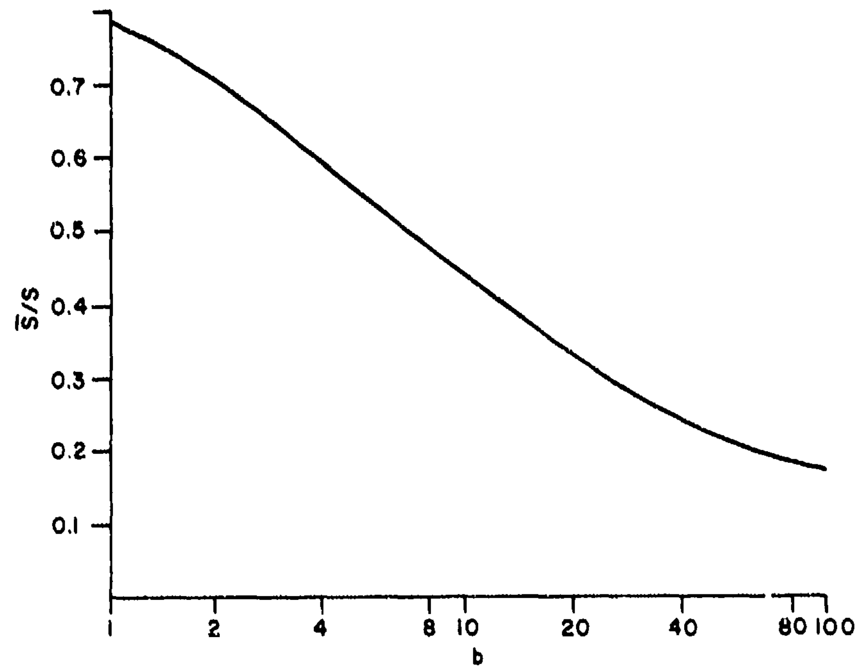

Fig. 2-8. Relationship between $\bar{S}/S$ and b [20].

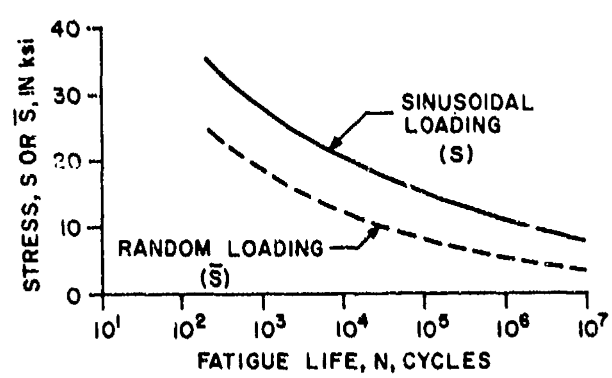

Fig. 2-9. Comparison of fatigue lives for random and sinusoidal loading [21].

The constants in Eq. (2-13) are fixed for a specific load (stress amplitude in a single-level test) and may vary for different values of load. Damage at failure is considered to be unity, which leads to an expression for damage as a function of cycle ratio:

$$D_i = \left(\frac{n_i}{N_i}\right)^{a_i}. \tag{2-14}$$

Equation (2-14) is of little value because it only provides information about the damage accumulated at one load level. When more than one load level is applied to a specimen, e.g., two varying loads, the order of load application becomes important and the damage expression of Eq. (2-13) must be generalized. Corten and Dolan performed the generalization by using the actual number of cycles at each load level and by assuming that damage accumulated at the lower level was influenced by the number of nucleation sites m caused by the higher level load. Because of the use of the actual number of cycles at a low load level instead of an equivalent number of cycles at the higher load level, it is necessary [24] for the exponent a to be independent of load magnitude. The use of a load-invariant exponent for the case of two different load levels leads to

$$N_f = \frac{N_h}{\lambda_h + R^{1/a}(1 - \lambda_h)}, \tag{2-15}$$

where

N_f = the number of cycles to failure
N_h = the number of cycles at the higher load
λ_h = the ratio of cycles at the higher load to the total number of cycles
R = the ratio of lower load to higher load crack propagation rates.

Equation (2-15) may be expanded to account for m sets of load history,

$$N_f = \frac{N_h}{\sum_{i=1}^{m} \lambda_i \left(\frac{r_i}{r_h}\right)^{1/a}}, \tag{2-16}$$

where i refers to load history i, subscript h refers to the highest load, r is defined in Eq. (2-13), and λ_i is the ratio of cycles at load i to the total number of cycles,

$$\lambda_i = \frac{n_i}{N_f}. \tag{2-17}$$

Corten and Dolan also asserted that a relationship existed between R and a ratio of lower stress to the highest stress,

$$R^{1/a} = \left[\frac{S}{S_h}\right]^d, \tag{2-18}$$

where the exponent d was determined experimentally and one value appeared to describe all the data for a single material and configuration in their experimental work. Incorporation of Eq. (2-18) into Eq. (2-16) leads to a generalized expression for life when several load levels are present,

$$N_f = \frac{N_h}{\sum_{i=1}^{m} \lambda_i \left(\frac{S_i}{S_h}\right)^d} . \tag{2-19}$$

The preceding development is based on an assumption that the damage nucleation cycle time is zero. Thus, all of the damage nuclei are created during the first load application, and the number of nuclei is a function of the largest load. Therefore, to apply the Corten-Dolan theory it is necessary to (a) identify the largest load, and (b) have a suitable value of the exponent d as determined by experiment.

When investigating 2024-T4 aluminum alloy, Swanson [25] found that the Corten-Dolan theory predicts conservative life for low prestress conditions and approaches correct life values at high prestresses. There are little data in the literature which provide values of the Corten-Dolan exponent d, a fact also noted by Gerks [26], and which limit the usefulness of this approach until values of d are experimentally determined and available.

It is possible to rearrange Eq. (2-19) into a form similar to that of Miner's hypothesis, Eq. (2-5), by using Eq. (2-17) and defining

$$\bar{N}_i = N_h \left(\frac{S_h}{S_i}\right)^d , \tag{2-20}$$

with the result that

$$D = \sum_{i=1}^{m} \frac{n_i}{\bar{N}_i} . \tag{2-21}$$

When reviewing damage at failure, that is $D = 1$, and comparing Eqs. (2-20) and (2-21), it is apparent that the Corten-Dolan theory can be interpreted as a Miner summation on a family of modified fatigue curves as defined by rewriting Eq. (2-20),

$$\bar{N}_i S_i^d = N_h S_h^d = \bar{C} = C S_h^{d-b} , \tag{2-22}$$

where $\bar{C}$ and C are constants selected to maintain the equality. The modified curve is obtained by passing a straight line through S_h as shown in Fig. 2-10.

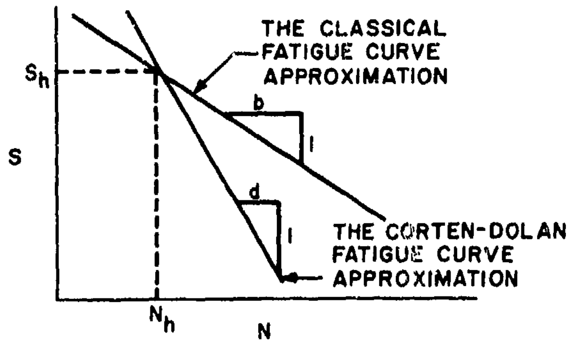

Fig. 2-10. Corten-Dolan modified fatigue curve.

The use of a modified *S-N* curve will allow the application of the Corten-Dolan theory to match that of Miner. There are two cases for which the Corten-Dolan approach is identical to Miner's: when $d = b$, and when the stress spectrum consists of monotonically increasing stress levels. In most cases, however, $d < b$ which gives a higher weighting to lower stress levels, or alternately, the predicted life would be shorter than the fatigue life predicted by Miner. Use of the modified *S-N* curve with Eq. (2-8) yields

$$S_e = \left[\frac{\sum n_i S_i^d}{\sum n_i} \right]^{1/d} \tag{2-23}$$

for an equivalent stress.

As in the case of Miner's theory, the Corten-Dolan expression for life can be extended to random load spectra.

The case of continuous spectra,

$$N_f = \frac{N_h}{\int_0^{S_h} \left\{ P[S] \left(\frac{S}{S_h} \right)^d \right\} dS}, \tag{2-24}$$

is an expansion of Eq. (2-19).

Shanley 1x and 2x Methods

Observing that fatigue crack growth tends to increase with crack depth and that reversed slip due to cyclic loading may cause atoms in a material to progressively unbond, Shanley [27] proposed an exponential relationship for crack growth,

$$h_i = A \exp(\gamma n_i) \ , \tag{2-25}$$

where
h_i = crack depth a load i caused by n_i cycles
A = a constant
γ = a factor dependent on load magnitude.

He further noted that the total strain ϵ caused by a load consists of both elastic and inelastic components,

$$\epsilon = \frac{S}{E} + CS^x, \tag{2-26}$$

of which only the inelastic strain contributes to crack growth. The exponent x was determined by curve fitting and represents the slope of a logarithmic S-N curve. Use of the above expression for inelastic strain to represent γ in Eq. (2-25) gives the following expression for crack growth:

$$h_i = A \exp(CS^x n_i). \tag{2-27}$$

The constant A was interpreted to represent an initial crack depth h_0 (i.e., let $n_i = 0$ in Eq. (2-25)), which leads to the Shanley "$1x$" theory defining crack growth,

$$h_i = h_0 \exp(CS^x n_i), \tag{2-28}$$

where C is a constant.

Rewriting Eq. (2-28) in terms of critical crack depth, which is considered to be a constant regardless of load amplitude, yields

$$h_f = h_0 \exp(CS^x N_i), \tag{2-29}$$

where h_f is the critical crack depth.

It can be seen that the S-N curve must be of the form

$$N_i = \frac{(\text{Constant})}{S_i^x} \ . \tag{2-30}$$

Thus, the Shanley theory predicts the nature of the S-N relationship as well as providing an approach to estimating cumulative damage. Damage is now defined as a ratio of h_i to h_f,

$$D_i = \exp\left[CS_i^x N_i\left(\frac{n_i}{N_i} - 1\right)\right] . \tag{2-31}$$

Shanley derived an expression for a reduced or equivalent stress which would yield identical damage over the same total number of load cycles as the actual load history. The expression for equivalent stress,

$$S_e = \left[\frac{\sum n_i S_i^x}{\sum n_i}\right]^{1/x} , \tag{2-32}$$

shows that all stresses in the spectrum, when raised to the x power, are weighted in proportion to their relative frequency of occurrence. The Shanley $1x$ theory, assuming a straight-line representation of the logarithmic S-N curve, will reduce to a statement of the Miner theory summation and is considered to be equivalent [19]. The primary difference between the Shanley $1x$ theory and the Miner theory is that Miner stated that damage was a linear function of cycle ratio, whereas Shanley merely required the existence of a damage relationship.

Shanley further noted that the $1x$ hypothesis did not seem to fit actual test data well at large values of n. As a result he developed what is called the Shanley $2x$ theory by assuming that initial crack depth was a function of the magnitude of the stress amplitude:

$$h_i = AS_i^x \exp\,(CS_i^x n_i) . \tag{2-33}$$

Use of Eq. (2-33) in the original derivation led Shanley to the $2x$ hypothesis. The $2x$ expression for equivalent stress gives greater weight to the damage which is accumulated at higher stress levels:

$$S_e = \left[\frac{\sum n_i S_i^{2x}}{\sum n_i}\right]^{1/(2x)} . \tag{2-34}$$

This approach tends to bring the $2x$ theory into closer agreement with test data. It should be noted that the exponent x used by Shanley and after which his hypothesis is named, is in fact identical to the exponent b from Eq. (2-3). That is,

$$S_e = \left[\frac{\sum n_i S_i^{2b}}{\sum n_i}\right]^{1/(2b)} , \tag{2-35}$$

which is identical to Eq. (2-34) with b substituted for x. The $2x$ theory (or $2b$) theory will predict earlier failure than Miner's theory.

When used for fatigue life predictions, S_e is determined by either Eq. (2-32) or Eq. (2-34), and x (or b) is derived from experimental data. The material S-N curve is entered at S_e to find the corresponding number of cycles N_e that may be applied to the specimen where the load causes stresses above the endurance limit. Fatigue life is predicted when N_e is multiplied by the total number of cycles and divided by the number of cycles above the endurance limit for the loading spectrum; that is,

$$N_i = N_e\left(\frac{n_i}{n_{ir}}\right) , \qquad (2\text{-}36)$$

where

n_{ir} = the number of cycles out of n_i cycles in which the load causes stresses above the endurance limit.

In a later elaboration [28] Shanley used data taken from rotating-beam tests of 2024-T4 aluminum to compute the mean and variation of the ratios of: (a) reduced stress (Eqs. (2-32) and (2-34)) to the observed stress at the same total life, and (b) cycles-to-failure at the reduced stress to observed life at the reduced stress. The results are displayed in Fig. 2-11 where the envelope of the test load sequence was either sinusoidal or exponential. For this analysis, the $2x$ method yielded ratios closer to unity than the $1x$ method. Very good results were obtained in predicting equivalent stresses; however, life predictions show a wide range.

The Shanley $2x$ hypothesis, Eq. (2-35) can be converted into linear summation form with a modified S-N curve in a manner similar to the manipulation of the Corten-Dolan hypothesis on pp. 14-18. In this case,

$$D = \sum_{i=1}^{m} \frac{n_i}{N_i^*} , \qquad (2\text{-}37)$$

where N_i^* represents the number of cycles to failure taken from a modified S-N curve defined by

$$N_i^* = \frac{N_i^2}{N_e} . \qquad (2\text{-}38)$$

The Shanley modified S-N curve is presented in Fig. 2-12. Comparison of the $-1/2b$ sloped line (Shanley $2x$ theory) with the $-1/b$ sloped line (Miner's theory) shows that stresses above S_e are given heavier weighting and stresses below S_e are

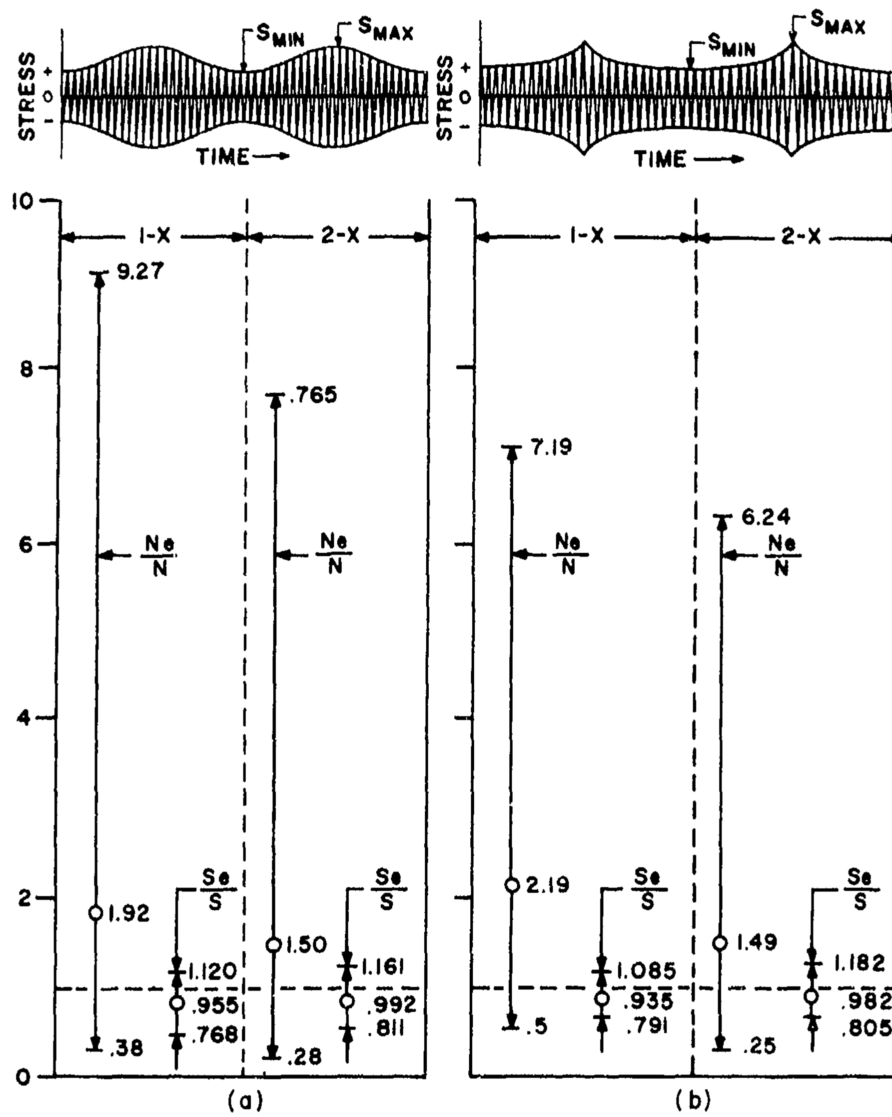

Fig. 2-11. Ratio of computed and observed S and N, taken from Shanley [28]. (a) Sinusoidal (140 tests) (b) Exponential (130 tests).

given less weighting for the Shanley theory than for Miner's theory. The pivot-point location depends on the value of N_e which (recall Fig. 2-11) is of poor accuracy. As a result the modified curve and Miner-type summation are of limited usefulness except to indicate the relative weighting of stress amplitudes.

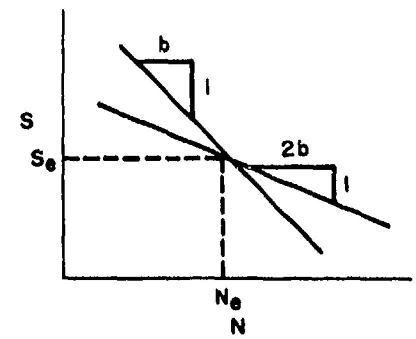

Fig. 2-12. Modification of a basic *S-N* curve for use in the Shanley 2*x* hypothesis.

2.3 Cumulative Damage Processes

In addition to the three cumulative damage theories which were covered in the previous section, there are several other theories which have been developed in an attempt to improve the accuracy of the time-to-failure prediction methods. These additional theories in many respects are similar to the primary theories previously covered. Several of them degenerate into a form identical to the Miner hypothesis. The superficial treatment given to these theories in the section does not mean that they are less important; it does mean that they have not been used extensively as a basis for vibration equivalence modeling.

The remaining cumulative damage theories have been classified as linear, nonlinear, and phenomenological.

Linear Cumulative Damage

The linear cumulative damage theories are based on experimentally determined *S-N* data which must be gathered or already exists for various combinations of material, temperature, geometry, and other suitable parameters as discussed in Section 2.1. *Linear* refers to the method of summing the fractions of consumed life and does not mean that the damage process itself is linear, a point which is sometimes misconstrued in the literature. A summary of linear cumulative damage theory is provided as Table 2-1.

Nonlinear Cumulative Damage

Nonlinear cumulative-damage theory, unlike linear theory, is the result of assuming that there is interaction between load history and damage. That is, the amount of damage at any load level depends on both the magnitude and number of occurrences of prior loads. These theories are usually valid whether or not it is considered that equal amounts of damage are caused by equal fractions of life for all stress amplitudes. A summary of nonlinear cumulative-damage theory is provided as Table 2-2.

Table 2-1. Linear Cumulative Damage Theories

Theory	*Features*	*References*
Miner	Discussed on pp. 11 to 14, Miner's damage summation is described by Eq. (2-5): $$D = \sum_{i=1}^{m} \frac{n_i}{N_i},$$ where D is damage, n_i is the number of cycles applied at load i, and N_i is the number of cycles to failure at the i load level.	7, 13, 14
Langer	Documented prior to the Grover theory, Langer presented the hypothesis that fatigue damage consisted of a two-stage process of crack initiation and crack growth.	7
Grover	Similar to the Langer approach; Grover divided the fatigue damage process into stages of crack initiation and crack propagation, and utilized definitive information about the damage process that is neither usually available nor easily measured. That is, $$N_f = N_i' + N_i ,$$ where N_f is the number of cycles to failure, N_i' is the number of cycles at load level i required for crack initiation, and N_i is the number of cycles required for a crack to propagate to failure.	29
Shanley 1x	Discussed on pp. 18 to 22, the Shanley theory results in an expression for fatigue damage which predicts the S-N curve and can be shown to match the Miner hypothesis. Recall the relationship of Eq. (2-30):	27

Table 2-1 (Continued)

Theory	*Features*	*References*
Shanley (Continued)	$$D_i = \exp\left\{CS_i^x N_i\left[\left(\frac{n_i}{N_i}\right) - 1\right]\right\},$$ where S_i is the stress amplitude at load i, and damage was defined as the ratio of crack length to critical crack length at failure.	
Lundberg	Describes the load history as a plot of varying stress vs number of load applications where the plot may be described as $$N_f = N_0 \exp(-\ell S_\nu),$$ where N_0 is the $S_\nu = 0$ intercept of a straight-line logarithmic S_ν vs N plot, S_ν is the varying stress, and ℓ is the slope of the loading spectrum curve. The resultant damage expression is $$D = \frac{N_0}{\alpha}\left[\ell^{-s}\Gamma(S-1)\exp(-\ell S_e)\right]$$ where α is the vertical intercept of the logarithmic $(S_\nu - S_e)$ vs N plot at $N = 1$, S_e is the endurance limit stress, and s is the slope. The number of cycles to failure is found by Miner's summation using the actual number of load cycles.	30
Valluri	A hypothesis based on dislocation theory and plastic deformation at the tip of a crack. Crack growth is assumed to exponentially increase with the number of stress cycles: $$h_i = h_0 \exp\left\{\left[\ln\left(\frac{h_f}{h_0}\right)\right]\frac{n_i}{N_i}\right\},$$ with h_0 determined using Griffith crack theory, and damage defined as	31

Table 2-1 (Continued)

Theory	*Features*	*References*
Valluri (Continued)	$$D = \frac{h_i - h_0}{h_f - h_0}$$ using h_f as the crack length at failure when $n_i/N_i = 1$.	
Manson, Freche, and Ensign	Divided the damage accumulation process into two stages: that of crack initiation, $$\sum \frac{n}{N_0} = 1,$$ where N_0 is the cycle life to initiate an effective crack; and for the crack propagation stage $$\sum \frac{n}{N} = 1,$$ where $$N = \Delta N + N_0$$ and $$\Delta N = PN^{0.6}.$$ P is a constant valued at 14 for SAE 4130 steel.	32
Sorenson	An approach to complete generality in developing a linear theory of isotropic cumulative failure, where the analytical model recognizes experimental observations and previous theory. The result is given as a time integral of a suitable invariant function of the stress tensor: $$D_{ij} = \int_0^t R(S_{ij})du,$$ where $R(S_{ij})$ expresses the instantaneous damage rate accumulation function, and failure occurs when $D = 1$.	33

Table 2-2. Nonlinear Cumulative Damage Theories

Theory	*Features*	*References*
Corten-Dolan	Damage summation is described by Eq. (2-13): $$D_i = m_i r_i n_i^{a_i} ,$$ which relates damage to the number of damage nuclei, rate of crack propagation, and an experimentally determined exponent.	22, 23
Shanley	Crack growth is expressed mathematically by Eq. (2-32): $$h_i = AS_i^x \exp(CS_i^x n_i) .$$	27, 28, 34
Henry	A complex procedure of *S-N* curve modification based on the damage caused by individual loads. Requires knowledge of the loading sequence, and a specialized mathematical model of the *S-N* curve which would fit only a narrow range of materials. That is, $$N = \frac{N_0}{(S_r - S_e)} ,$$ where the variables are defined as in the Lundberg theory described in Table 2-1.	35
Poppleton	An analytical theory based on the Corten-Dolan theory [23] and work by Torbe [36] for the case of a stationary Gaussian stress history; no experimental verification.	37

Phenomenological

A summary of phenomenological fatigue damage accumulation theory is provided as Table 2-3. This category of cumulative-damage theory includes hypotheses which may be either linear or nonlinear. The term *phenomenological* was applied because the theories were based on observation and various attempts to (a) describe failure boundaries based on those observations or (b) use the observed information to modify basic S-N curves.

Table 2-3. Phenomenological Theories

Theory	*Features*	*References*
Kommers	Kommers assumed that damage is a function of both cycle ratio and the stresses associated with varying loads. This concept was furthered by the work of Richart and Newmark, and Marco and Starkey. The latter suggested that the shape of the damage curve was defined by $D_i = \left(\frac{N_i}{N_f}\right)^\gamma$ where γ is a stress-level-dependent exponent. Application of this method requires specialized bilevel loading test data which are not normally available.	16, 38, 39
Freudenthal and Heller	Introduced a stress interaction factor to create an expression for a fictitious S-N curve. The authors assume that $I = \frac{N_i}{N_i^*}$ defined an observed stress interaction factor I, and that N_i^* is the expected life at load i as read from the fictitious S-N curve. The concept of linear cumulative-fatigue damage was applied using the fictitious S-N curve with the following result: $D = \sum \frac{In_i}{N_i}$.	17, 40

Table 2-3 (Continued)

Theory	*Features*	*References*
Levy	A fatigue-life prediction procedure using empirical constants in each step of the loading sequence, in the form $$\log N_f = a \log\left(\frac{n_1}{\sum n_i}\right) + b \log\left(\frac{n_2}{\sum n_i}\right) + \ldots + M,$$ where a, b, etc., and M are the empirical constants and are functions of N_1, N_2, etc. Note that the increasing index, 1, 2, etc., refers to increasing levels of load. Use of this approach requires the collection of a significant volume of specialized test data.	41
Head and Hooke	Suggested a cumulative-fatigue damage rule for structures under random load using measured life data from discrete loading: $$N_{\bar{S}} = \left\{ \sum_{i=1}^{m} \frac{X_i \left[\exp\left(\frac{-X^2}{2}\right)\right] \Delta X}{N_i} \right\}^{-1},$$ where $N_{\bar{S}}$ is the number of equivalent random cycles, and $X = S/\bar{S}$.	42
Eshlemen et al.	Presented a cumulative-damage expression for the life of a two-degree-of-freedom structure under random loading: $$N_{\bar{S}} = \left[\int_0^{\infty}\int_0^{\infty} \frac{P(x,y)dxdy}{N_{x,y}}\right]^{-1}$$ and	43

Table 2-3 (Continued)

Theory	*Features*	*References*
Eshleman (Continued)	$$D = \sum\sum \frac{n_{x,y}}{N_{x,y}},$$ using a Rayleigh probability distribution to describe stress peaks in both the x and y coordinates.	
Parzen	A theoretical cumulative-damage model based on level-crossing processes was developed where the random damage and random input problems were both included. He assumed that damage is a nonnegative random variable associated with the application of load; that is, $$D = \sum_{i=1}^{m} D_i(S)$$ for nonnegative identically distributed variables. Thus $$E[D] = \sum \left\{ E\left[\sum_i n_i\right] E[D_i(S)] \right\},$$ which is a very general statistical model. Parzen's theory provides a stationary random model of the number of stresses exceeding some level, with which the damage per cycle may be estimated.	44
Gatts	An early attempt to base fatigue failure on strength using stress-strain relationships that account for the change of material properties with vibration history. The stress-strain hysteresis loop was used as the phenomenological basis, damage was measured as a reduction in the endurance limit and	45, 46

Table 2-3 (Continued)

Theory	*Features*	*References*
Gatts (Continued	failure stress, and no damage was assumed to occur at stress levels below the endurance limit. That is, $$D = 0 \quad \text{for} \quad S \leqslant S_e \ .$$ Also, $$\begin{cases} n = 0, & S_e = (S_e)_0 \\ n = N, & S = S_{f,N} \ . \end{cases}$$ Gatts' theory presented a nonstationary deterministic model based on hysteresis loop area, assuming some given relationship for the reduction in strength (damage) as a function of vibration history.	
Kozin and Sweet	Created a general theory of failure based on the work of Parzen [44] and Gatts [45]. Damage was assumed to be a nonstationary random function of the irreversible work input for which the average damage per cycle was empirically established. The parameters involved are the stress-strain hysteresis loop area, number of cycles to failure, and rate of change of hysteresis loop area as a function of stress level. The damage after j cycles of sinusoidal load is $$D_i = \sum_{j=1}^{m} D(A_j)_i \ ,$$ where $D(A_j)_i$ is the random damage due to load cycle j at load level i. The fundamental expression of the Kozin theory is	47, 48

Table 2-3 (Continued)

Theory	*Features*	*References*
Kozin (Continued)	$$E\left[\frac{D(A_j)_i}{K^*}\right] = \left\{E\left[\sum_{j=1}^{N_i}\left[\frac{(A_j)_i}{(A_1)_i}\right]\right]\right\}^{-1},$$ where K^* is a constant depending on the material, N_i is the number of cycles to failure at load level i, and $(A_j)_i$ is the hysteresis loop area after load cycle j at load level i.	

CHAPTER 3
CUMULATIVE-DAMAGE EQUIVALENCES

3.1 Preliminary Considerations

Vibration equivalence based on cumulative-damage theory is assumed to exist when two or more vibration experiences produce like amounts of damage in a given specimen. All vibration experiences are assumed to cause damage, and the relationship between vibration experience and damage must be known or assumed.

The techniques of fatigue-life prediction and cumulative-damage equivalence both draw on the concept of damage accumulation; however, there is a fundamental difference in objective. Fatigue theory is used to develop accurate life predictions, whereas equivalence theory is used to compare the damage resulting from different vibration experiences. As a result, the use of those equivalence techniques based on the cumulative-damage concept to not relieve the designer from his responsibility to verify that a specimen has adequate service life. The objective of cumulative-damage equivalence theory is to provide techniques by which test time and test type transformations may be accomplished once a specimen is known to have adequate life.

This chapter provides a review of the cumulative-damage equivalence techniques. It was indicated earlier that the cumulative-damage equivalences are valid when a predictable relationship may be defined between load amplitude and the number of load applications that produce failure. For fatigue processes the prediction relationship is an appropriate *S-N* curve. For the wearout processes it is another relationship which may have characteristics similar to the characteristics of the fatigue process relationship.

The most valuable aspect of the cumulative-damage equivalence techniques is that they provide a method to change test time as a function of vibration amplitude. They are also valuable as a design tool which may be used to form a comparison between two or more different vibration experiences on a damage basis.

Damage Mechanisms

Two types of damage are modeled by the cumulative-damage equivalence methods. The first type of damage is material fatigue damage. If the application of suitable failure criteria to a specimen indicates that structural fatigue life is the limiting design parameter, then the cumulative-damage equivalence techniques may be used directly. The second type of damage is wearout. Wearout damage results from those mechanisms, such as wear, friction, and fretting, which consume some usable portion of specimen life and which may be attributed to vibration of the specimen. Other wearout processes such as corrosion, weathering, radiation damage, chemical change, and abuse, are consigned to the problem of specimen life prediction.

Vibration-induced wearout damage will appear in one or a combination of three forms.

First, a wearout process may be level sensitive and cause damage only above a defined load threshold. An example would be a bolted connection in which friction prevents relative motion between the joined elements until a defined load threshold is exceeded. Threshold wearout processes are usually treated by summing the damage produced by those load excursions which exceed the damage threshold.

Second, a wearout process may be sensitive only to the number of load repetitions, regardless of load amplitude, in a range of load amplitudes that bound the expected service or test load conditions. Certain types of wear and fretting fall into this category. Repetition wearout equivalence exists when the number of load applications under one set of circumstances equals the number of load applications under another set of circumstances.

Third, a significant number of damage-producing wearout processes have characteristics which imitate the material fatigue processes. These general wearout processes occur when there is a definite relationship between load amplitude and the number of load applications that produce specimen failure. This wearout process finds application in those instances where a complex specimen cannot be accurately modeled, yet it is known or suspected that a load-vs-life relationship exists. The actual damage mechanism is undefined, although it may be entirely wearout or a combination of wearout and fatigue. Once the general wearout process for a specimen is characterized by a load vs N relationship, the wearout relationship is used to form a vibration equivalence in an identical manner to the fatigue processes.

Information on the general wearout process does not appear frequently in the literature. Harris and Crede [49] indicated in 1961 that failure curves for electronic equipment subjected to varying levels of vibration are similar to the S-N curves for metals.

Figure 3-1 provides a summary of the cumulative-damage processes which are used to form vibration equivalence.

Load-vs-Life Relationships

A load-vs-life relationship is an expression of the fatigue or wearout life of a specimen as a function of load magnitude. An example of a fatigue load-vs-life relationship for a simple specimen would be an S-N curve which was derived from rotating-beam test data. Fatigue data in the form of S-N curves are distributed throughout the literature and are relevant to many specialized combinations of material, specimen configuration, and loading technique. Most of the available fatigue data are of the rotating-beam type, whereas fatigue data under axial loading are frequently required and are of more interest [50]. The prime

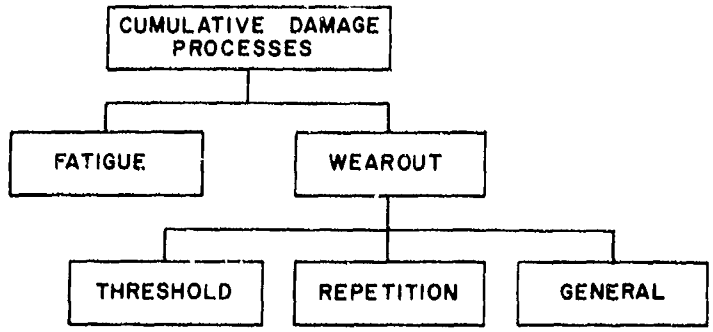

Fig. 3-1. The cumulative-damage processes.

characteristic of a load-vs-life (or S-N) relationship, which is used in forming a cumulative-damage equivalence, is the slope of a plot of the relationship made to logarithmic scales. The reader is cautioned that the selection of an appropriate slope involves the consideration of several factors.†

Although fatigue damage is caused by every stress cycle, the majority of damage has been found to accumulate in narrow frequency bands centered on the specimen resonant frequencies [49]. As a result of this fortunate occurrence, it is possible to simplify the formulation of vibration equivalences by considering only the damage accumulated at the specimen resonances.

For a single degree of freedom (SDF) linear system, that is, where stress is proportional to load by a constant ratio, specimen life may be described in terms of either specimen response or specimen excitation. Assuming that an applicable S-N relationship exists which has a constant slope b over the range of stress amplitudes of interest, Eq. (2-3) may be restated in terms of specimen response. Thus

$$NS^b = Tf_i(A\hat{V})^b = C_1 \ , \tag{3-1}$$

where T is the time to failure at a specimen's response acceleration amplitude $\hat{V}$, f_i is the response frequency, and A is the constant of proportionality between $\hat{V}$ and stress.‡ In practice the following form is often more convenient:

$$T\hat{V}^b = C_2 \ , \tag{3-2}$$

where C_2 must be found experimentally.

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† As described in Chapter 2 the S-N curve is influenced by the type of load (torsion, bending, tension), the ratio of alternating to mean stress, temperature, stress concentrations, the degree of stress reversal, and perhaps the cyclic rate of loading.

‡ As an example consider a simple cantilever beam excited by support motion. The maximum bending stress is found from $S = Me/I = A\hat{V}$, where M is developed by considering inertial forces. Thus $M = \bar{m}\hat{V}g\bar{y}$, where $\bar{m}$ represents the beam mass concentrated at the center of gravity and located $\bar{y}$ distance from the foundation end of the beam. It is now possible to write the desired expression $A = \bar{m}g\bar{y}e/I$.

It is possible to restate Eq. (2-3) in terms of the specimen excitation acceleration amplitude V, instead of the specimen response acceleration amplitude $\hat{V}$. Thus

$$NS^b = Tf_i[AH(f_i)V_i]^b = C_1 \ , \tag{3-3}$$

where the subscript i refers to the conditions at frequency i, and $H(f_i)$ is the specimen response amplification factor at f_i. Equation (3-3) may be specialized for the linear SDF specimen excited at resonance,

$$Tf_n(AQV)^b = C_1 \ , \tag{3-4}$$

where Q represents the specimen resonant transmissibility.

Additional yet necessary complexity is introduced when a proper $H(f_i)$ must be selected for use in Eq. (3-3). The additional complexity arises when $H(f_i)$, or perhaps Q in Eq. (3-4), is not constant but varies with V in addition to frequency. For example, it is often observed that the measured amplification ratio of a specimen decreases with increasing V. This change in specimen response is caused by specimen damping and stiffness properties. The effects of damping and stiffness will be reviewed; however, it is important to note that an amplification factor which was found experimentally may be conservatively viewed as constant over a narrow range of increasing V,† perhaps up to $2V$ or greater depending on the specimen.

Another load-vs-life relationship was described in the previous section as the general wearout damage process. This process is characterized by a relationship between either excitation or response acceleration amplitude and N. The V-N curve for a specimen, assembly, or system would be formed using data taken from service history or generated under controlled laboratory test conditions. The actual failure mechanism may change as a function of acceleration magnitude and excitation frequency. That is, several failure mechanisms are usually present in a complex specimen. Only one of them is the primary source of failure under a given set of excitation (response) amplitude and frequency conditions.‡

General wearout damage data are usually more specialized than fatigue damage data because the actual damage mechanism may be unknown. As a result the data are valid for a single configuration of a specimen, and the data may not be generalized to encompass a group of similar specimens unless sufficient data exist to support the generalization. A V-N relationship based on excitation amplitude is needed for each predominant frequency-dependent failure mechanism.

†The use of a constant amplification factor with decreasing V must be questioned, as $H(F_i)$ may increase as V decreases.

‡At a given excitation frequency a specimen may accumulate fatigue damage. At ano[illegible] excitation frequency the same specimen may be subject to wear or fretting. A d[illegible] damage accumulation criteria would apply at each frequency.

The formulation of a general wearout equivalence would involve a damage summation using as many individual V-N relationships as are needed at each significant response frequency over the frequency range of interest.

A $\hat{V}$-N relationship based on response amplitude may be found by direct measurement in the laboratory and used to form a vibration equivalence. A $\hat{V}$-N response amplitude relationship is developed in two steps. The first step is to select one or more locations on a specimen where specimen motions reflect specimen excitation over the entire frequency range of interest. The second step is to correlate N with the measured response levels. The resultant response amplitude $\hat{V}$-N relationship is used to characterize the specimen, and equivalence between different excitation spectra may be based on specimen responses to the different spectra.

3.2 Test Time Scaling

Vibration-equivalence time-scaling techniques provide the environmental engineer with a valuable tool which may be used to decrease test time. These techniques are also used to form a comparison of the relative severity of different vibration experiences by scaling each vibration record to the same time duration. In addition, for the situation in which a distribution of vibration levels is encountered in the field and it is desired to simulate the field vibration by a single-level test, the environmental engineer may use the time-scaling techniques to design a test at the maximum field level. Such a test would provide field vibration simulation and avoid level enhancement.

The time-scaling techniques are presented with the presupposition that the slope b of an appropriate S-N curve is known. It shall be assumed for simplicity that if a wearout process is used instead of a fatigue process, the slope of an appropriate load-vs-N curve may be substituted for the exponent b in each relationship.

The time-scaling practices were established by maintaining constant cumulative damage between various tests. The resultant time-scaling techniques are thus valid for maintaining constant fatigue damage, and the application of these techniques does not insure equality among other parameters of the specimen. For example, if an accelerated vibration test, i.e., a test performed at higher excitation levels for the purpose of reducing test time, were applied to a mechanism, the mechanism might not function properly during the accelerated test. Such a circumstance is not a limitation, providing the tested mechanism is not damaged by the higher level excitation. The environmental engineer may elect to test the example mechanism in two steps. First, a reduced-time, high-excitation-level test would be performed which determines the ability of the specimen to endure the accumulated damage. Second, a test would be performed at nominal excitation levels (usually following the endurance test), during which the mechanism would be energized and its performance evaluated. Such a situation is depicted in Fig. 3-2. Curve a represents a stress (load) level which will cause a temporary

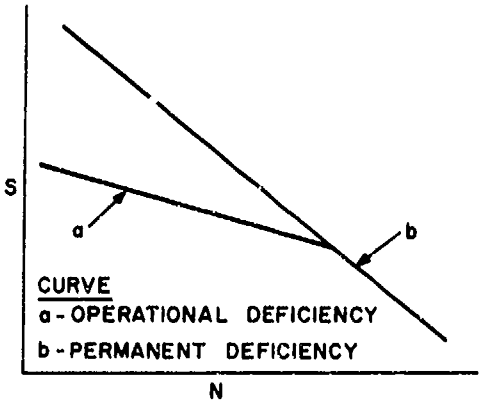


Fig. 3-2. Specimen operational limitations.

functional degradation of the specimen, whereas curve *b* represents permanent degradation. The endurance test would be performed anywhere in the region below curve *b*, and the performance test would be performed at levels below curve a; all to the left of the intersection of the two curves. A two-level test would be unnecessary in the region to the right of the intersection.

In some instances the environmental engineer may not be able to specify a vibration test level which will achieve a desired reduction in test time. The higher excitation level would be unachievable if the specimen fragility level were below the desired acceleration excitation level. When the excitation level of a specimen is limited, the environmental engineer may choose one or more of the following options:

1. Test at a lower excitation level and accept the increase in test time.
2. Allow the sensitive component to fail during the high-level test and refurbish prior to low-level performance testing.
3. Run the high-level test with the sensitive component replaced by a dynamically similar dummy component which is not level sensitive. The dummy component would be replaced by an actual component prior to low-level performance testing.

Options (2) and (3) are valid approaches if sufficient data are available to demonstrate that the level-sensitive component will withstand the total unaccelerated vibration history at its location on the specimen.

The following discussion covers the scaling of test time in which the type of test is held constant. Specific test types covered are (a) sine dwell to sine dwell, (b) sine sweep to sine sweep, (c) multiple sine dwell to multiple sine dwell, and (d) random to random. Changes in test type, e.g., sine sweep to sine dwell, etc., are covered in section 3.3.

Simple Spectrum Equivalence

The conversion of a single-frequency, sinusoidally varying load at one load amplitude to another load amplitude at the same frequency is the least complex equivalence computation. However, when performing this computation the environmental engineer must recognize the existence or nonexistence of damping and specimen response linearity characteristics.

Two primary assumptions are inherent in the procedure for changing the test time scale. First, it is assumed that the most critically stressed area has been located and that the stresses in this area can be estimated with suitable accuracy. Second, it is assumed that a single *S-N* curve exists which will characterize the most critically loaded (stressed) area of the specimen.

Sinusoidal Dwell Equivalence. Starting with a linear, undamped, SDF specimen where the critical stress S_c is related to specimen response $\hat{V}$, the application of Eq. (3-2) yields

$$T_1 \hat{V}_1^b = C_2 = T_2 \hat{V}_2^b$$

or (3-5)

$$\hat{V}_2 = \hat{V}_1 t_R^{1/b},$$

where the subscripts 1 and 2 denote the original and sought response levels, respectively, and the relationship

$$t_R = \frac{\text{(original time)}}{\text{(sought time)}} = \frac{t_1}{t_2} = \frac{T_1}{T_2} \tag{3-6}$$

represents the desired ratio of original vibration time to test time. The time ratio t_R is now seen to hold the same relationship between the original and sought time durations as exists between specimen life at the corresponding stress levels. The item T is determined by dividing the number of cycles-to-failure at the corresponding response level stress by the test frequency. That is,

$$T_1 = \frac{N_1}{f_1} \text{ and } T_2 = \frac{N_2}{f_2},$$

or, when the frequency is held constant,

$$t_R = \frac{N_1}{N_2},$$

which is the case for simple time-scaling equivalence.

The application of either Eq. (3-3) or (3-4), which relate specimen critical stress and specimen excitation, to the same linear, undamped, SDF specimen, provides

$$V_2 = V_1 t_R^{1/b}, \tag{3-8}$$

which is identical to Eq. (3-5) except that the former is expressed in terms of specimen response, whereas the latter relates the excitation levels.

Damping. Damping due either to the properties of the specimen materials or to the techniques of specimen construction causes an observed reduction of relative response amplitude with increased excitation amplitude. Damping is classically described as viscous, Coulomb, or a combination thereof. Viscous damping occurs when the amount of energy absorbed by the specimen is directly proportional to the relative velocity between the responding portion of the specimen and the portion of the specimen which is being acted upon by a force. Coulomb damping is frequently referred to as friction damping and the energy absorbed is proportional to the relative displacement between the responding and excited portions of the specimen.

The type of damping may be determined experimentally by plotting the successive displacement swings of the specimen, following excitation, when the motion is allowed to subside on its own accord. An exponential decrease in the magnitude of the swings indicates that only viscous damping is present, whereas a linear decrease indicates that only Coulomb friction is present. When both forms of damping are present the plot will at first show an exponential decrease, with the decrease becoming linear as the motion subsides. It is interesting to observe that the natural frequency of a free vibration with viscous damping is reduced by a small amount, whereas Coulomb damping causes no change in the system's natural frequency.

The effects of viscous damping on an SDF system are easily visualized by plotting the ratio of response acceleration to excitation acceleration (or response displacement to excitation displacement) against the ratio of excitation frequency to specimen natural frequency. Such a plot is given as Fig. 3-3 where ζ represents the ratio of specimen damping to critical damping.

The presence of Coulomb friction greatly complicates the analysis of the steady state response of a specimen as produced by an alternating force, especially if the frictional forces are large enough to cause halting of the relative motion between the forced and responding elements of the specimen. In general, when the frictional forces are small compared to the applied forces, continuous motion will result with the response amplitudes theoretically approaching infinity at resonance. It was shown by Den Hartog [51] that the critical ratio of frictional forces to excitation forces is $\pi/4$, That is, when the ratio of frictional forces to excitation forces exceeds the value $\pi/4$, discontinuous motion will result and halting will occur. The reader is directed to Den Hartog's work for a detailed treatment of this subject.

The general qualitative characteristics of material damping have received considerable attention, with results which are important to the formulation of vibration-equivalence techniques [49]. In particular, Lazan [52] related the resonant amplification of a specimen to the strain energy U_s and dissipated energy U_d under forced vibration as

$$Q = \frac{U_s}{U_d} . \tag{3-9}$$

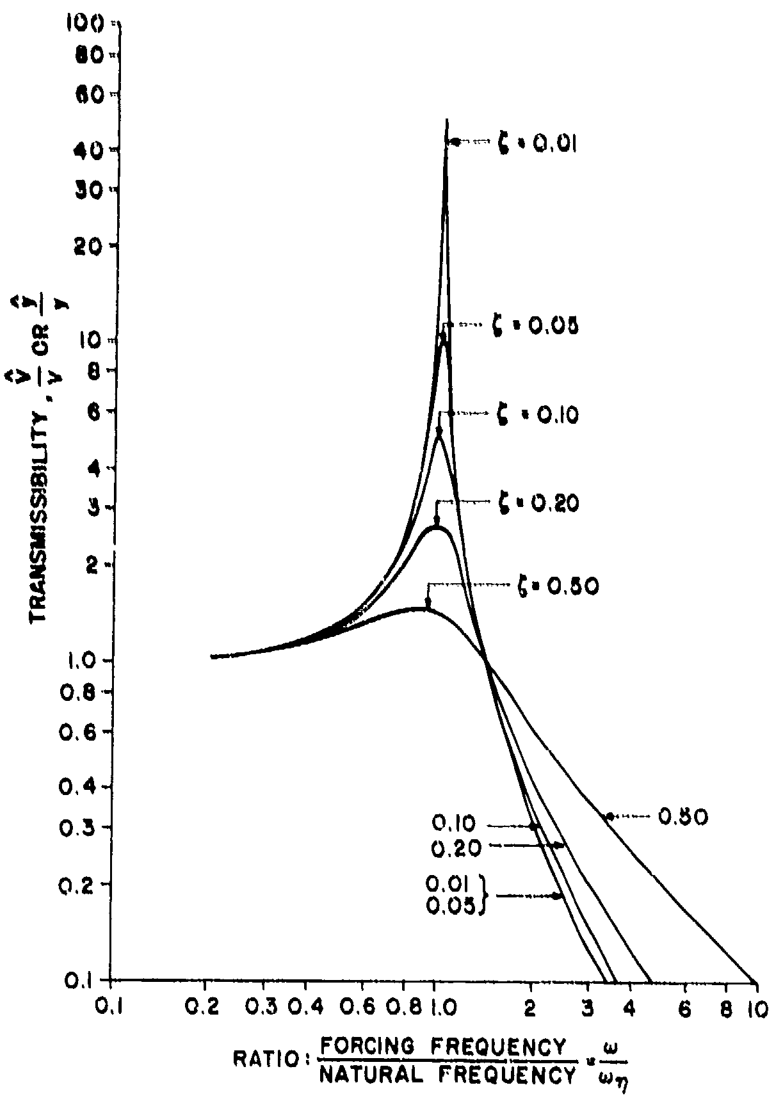


Fig. 3-3. Transmissibility functions for an SDF system.

The strain energy is proportional to the square of stress, and thus it is proportional to the square of vibration response. The energy dissipated in the specimen due to damping is also related to the stress level as

$$U_d = CS^n, \tag{3-10}$$

where the factor C is constant for a given material. The data generated by testing a large variety of structural material were tabulated and plotted. These data were found to lie within a definite region, as shown in Fig. 3-4. A plot of a "typical" viscoelastic adhesive material data and a plot of a large plastic-strain damping material data were included to show the effects of large damping. A structural material is one which does not have significant plastic-strain damping, nor significant magnetoelastic damping. It is important to observe that the structural materials exhibit a mean slope or $n = 2.4$ up to stress levels which represent 80% of the conventional endurance-level stress. At this point the value $n = 8$ more nearly represents the mean of the data distribution.

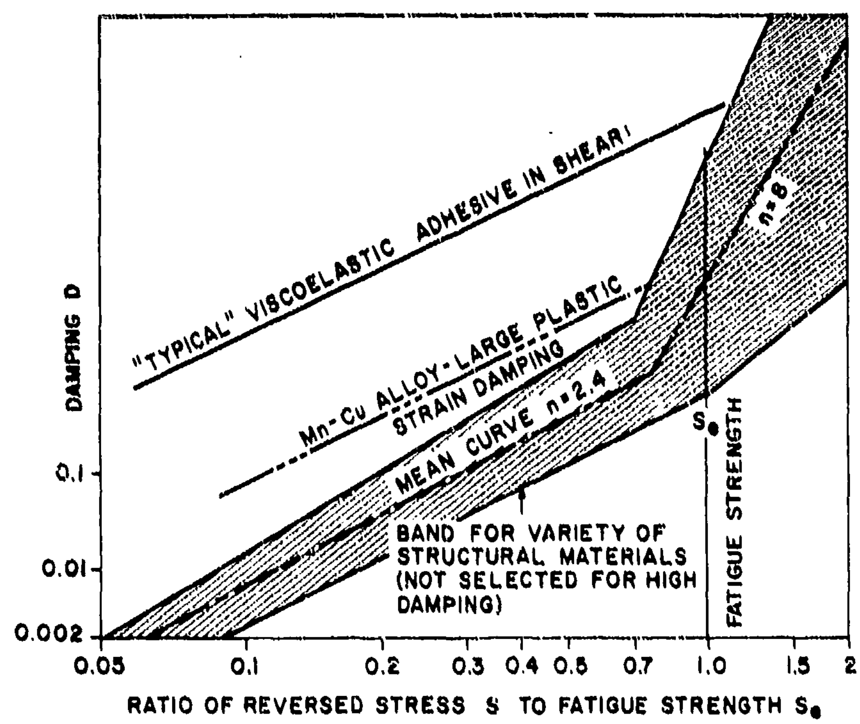


Fig. 3-4. Range of damping properties for a variety of structural materials.

Equation (3-9) may be rewritten as

$$Q = \hat{A}[\hat{V}]^{2-n}, \tag{3-11}$$

where the constant $\hat{A}$ is sensitive to a specific set of geometric, material, and stress-pattern factors for a specimen. As a result, $\hat{A}$ will assume different values for each response frequency of a specimen, a fact which indicates that there may be no constant relationship between Q and the response frequency unless the specimen is viscoelastically damped. In this case Q is a function of $\hat{A}$ only, a factor important in the selection of sweep rate in sinusoidal sweep testing.

The effects of structural damping become clear when Eqs. (3-4) and (3-11) are combined:

$$V_2 = V_1 t_R^{(n-1)/b}, \qquad (3\text{-}12)$$

where the value of n may vary from 2 to 8, with $n = 2.4$ being the preferred value for structural materials when the stress levels are no more than 80% of the endurance stress.

Linearity. As suggested at the beginning of this section, specimen response linearity characteristics will have an effect on the formulation of an equivalence. A nonlinear specimen will exhibit changes in response not related to excitation levels by a constant ratio, irrespective of damping. For solutions to the linearity problem we may look to nonlinear vibration theory as a source of techniques to predict the behavior of a specimen. Such theory will provide exact solutions for certain specialized cases involving free vibration; however, exact solutions for the forced vibration of nonlinear specimen are virtually nonexistent. An exception would be those specimens which can be represented by a stepwise linear model.

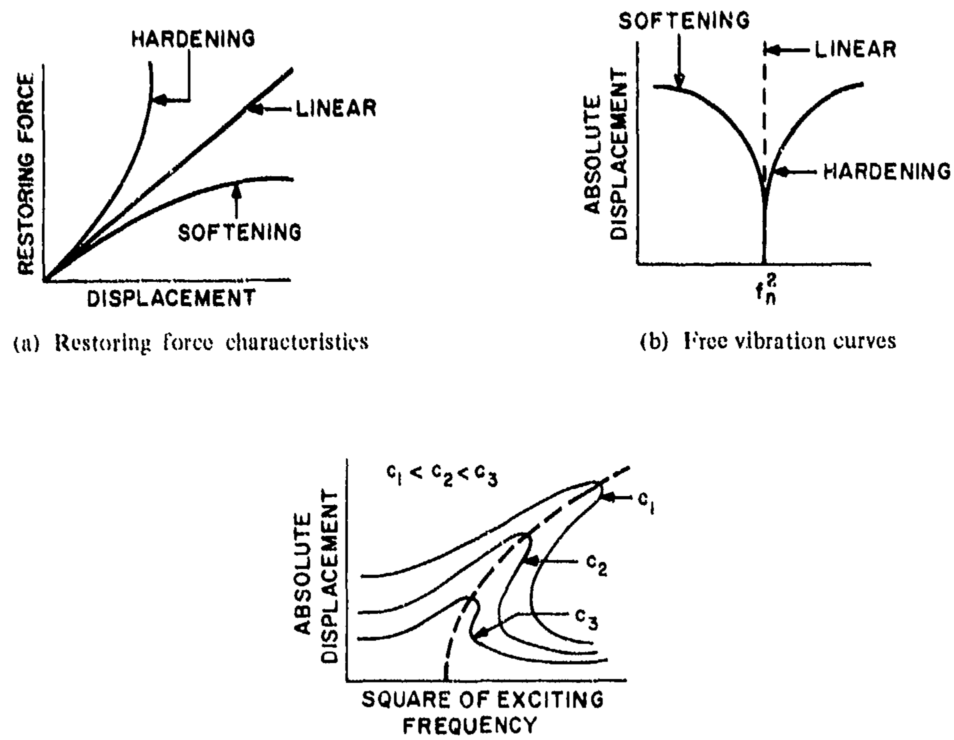
The two fundamental nonlinearity models are the hardening model and the softening model. These models are illustrated by Fig. 3-5a. The hardening model is one which tends to resist displacement with a restoring force which increases more rapidly than the displacement. The reverse situation is called the softening model. The effects of hardening and softening on the freely vibrating specimen are shown in Fig. 3-5b. In Fig. 3-5b it is seen that the system resonance frequency decreases with increasing displacement for the softening model, and increases with increasing displacement for the hardening model. A hardening system with damping is shown in Fig. 3-5c. As in the case of the linear system, damping defines the resonance response amplitude.

The effects of specimen nonlinearity are often observed in the vibration laboratory. For example, a specimen may be subjected to a low-excitation-level, sinusoidal-sweep vibration for the purposes of recording resonant response frequencies. Next, the same specimen may be subjected to a higher excitation level, sinusoidal dwell at one resonance for evaluation purposes. If the specimen is nonlinear, it will be observed that the excitation frequency which produces maximum response will no longer coincide with the resonance frequency determined by the low-level sinusoidal-sweep test.

The occurrence of specimen nonlinearities, when recognized, may be accounted for by one of two approaches. First, the specimen stress vs excitation amplitude may be experimentally recorded while the excitation frequency is held constant. In this case the vibration equivalence would be formed on a stress basis directly,

$$S_2 = S_1 t_R^{1/b}. \qquad (3\text{-}13)$$

The second approach would be to adjust the excitation frequency to match the maximum response frequency of the specimen and to scale the test time accordingly. In this case, t_2 must be corrected and the scaled test duration would be computed as



(a) Restoring force characteristics

(b) Free vibration curves

(c) A hardening system with damping

Fig. 3-5. Nonlinear characteristics.

$$\hat{t}_2 = t_2 \frac{f_1}{f_2} , \tag{3-14}$$

where $\hat{t}_2$ is the corrected test duration, f_2 is the response frequency at the higher excitation level, and f_1 is the response frequency determined by the lower excitation level test.

Sinusoidal Sweep Equivalence. The next most simple time-scale equivalence is performed on the sinusoidal sweep test. That is, to effect a change in test time by varying the number of sinusoidal sweeps and the test excitation level. It is assumed that the specimen is an SDF system and that the original sinusoidal sweep test was properly designed [53]. The duration and level of a sinusoidal sweep test are scaled as in the case of the sinusoidal dwell test, the exception being that t_R is found by the ratio

$$t_R = \frac{\text{original number of sweeps}}{\text{desired number of sweeps}} . \tag{3-15}$$

Further parametric manipulations may be possible, that is, changes in sweep rate; however, such manipulations are complex and involve several variables. A certain number of cycles of excitation is required at resonance to allow the specimen to develop full steady state response. A variation of excitation frequency, as in a sinusoidal sweep test, produces a given number of cycles in the region of each resonance. Each region of resonance is usually characterized by the half-power bandwidth B, as shown in Fig. 3-6. Full steady state response of a specimen subjected to a sinusoidal sweep test may be expected only if the sweep rate is sufficiently slow as to allow a full response buildup within B. Such factors as sweep direction, sweep rate, natural frequency, sweep method, and damping have importance in determining specimen response to swept excitation. One approach, as proposed by Cronin [54], provides a good approximation of maximum steady state response in terms of a sweep parameter s. If X represents the ratio of resonance response buildup to steady state response, then

$$X = 1 - \frac{1}{(\exp 2.86 s^{-0.445})}, \tag{3-16}$$

where

$$s = \frac{\hat{f}}{B^2} = \frac{Q^2 \hat{f}}{f_n^2} \tag{3-17}$$

and $\hat{f}$ is the absolute value of the time rate of change of frequency through B. In addition, the sweep excitation causes a specimen to achieve maximum response at higher frequencies on increasing frequency sweeps, and maximum response at

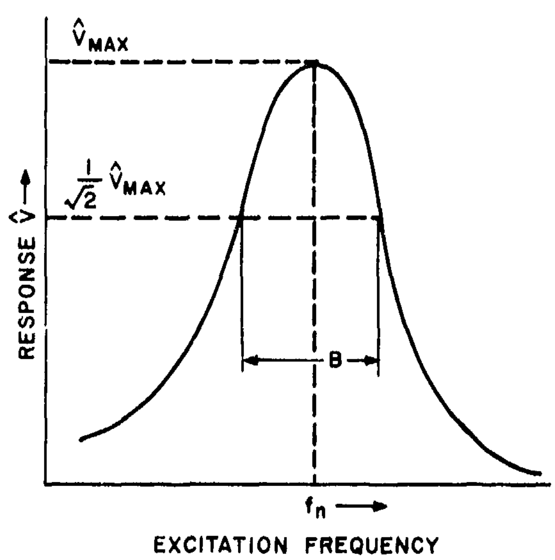


Fig. 3-6. Half-power bandwidth B defined.

lower frequencies on declining frequency sweeps; the amount of offset being a function of the magnitude of $\hat{f}$ [55]. A method which is used to change sweep rates is discussed in Section 3.3.

Complex Spectrum Equivalence

The conversion of a field or laboratory vibratory excitation which consists of several discrete sinusoidal excitation frequencies, or one or more random excitation spectra, defines the next level of complexity in test time scaling.

Several Sinusoidal Excitations. The excitation of a specimen by a sequentially or simultaneously applied group of single-frequency sinusoidal excitations may be used to represent field vibration conditions. The field excitation spectra may encompass a wide range of excitation frequencies. However, the specimen will tend to respond to this excitation within narrow frequency bands, each centered about a resonance frequency. Although fatigue damage accumulates at all stress cycles, the error in assuming that damage occurs only within the half-power bandwidth B is less than 3%. As a result, the time scaling of a field or test condition is carried out considering only the damage accumulated at each resonance.

A usual assumption which accompanies the following equivalence practices is that each resonance acts independently of all other resonances. This is a reasonable assumption provided that sufficient frequency separation exists between the response frequencies. If there is insufficient frequency separation, the influence of one response on the other may appear as shown in Fig. 3-7. This figure shows a typical response acceleration vs frequency plot for a specimen which has two resonances with significant interaction. The solid line indicates the measured response. The dashed lines show the responses at each resonance, where the specimen was allowed to respond independently. The distortion of B is an important source of error for the sinusoidal sweep test (recall Eq. (3-17)). However, this resonance coupling introduces very little error in the simple sinusoidal-dwell time-scale equivalence computation.

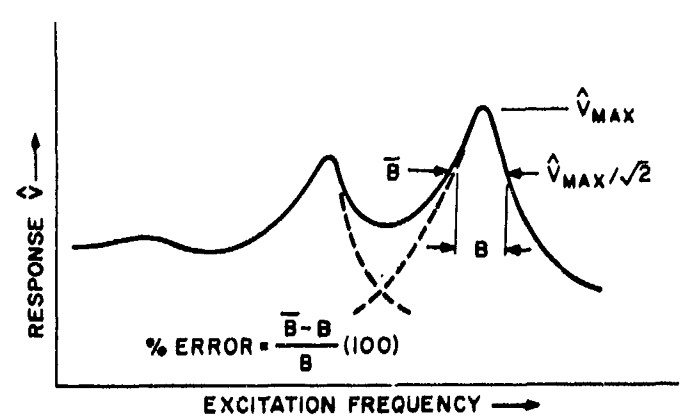


Fig. 3-7. Distortion of B due to adjacent resonance.

The time-scale modification of a field or laboratory excitation record, when that record is composed of more than one discrete frequency of excitation, consists of scaling the excitation bond separately at each frequency, as described on pp. 38 to 46. The resultant equivalent test would be described by a revised set of excitation levels and a corresponding excitation time at each level.

A variation of excitation level at any one discrete frequency is frequently observed in the field. Time-scale modifications may be accomplished by computing an effective stress S_c and then performing a time-scale modification based on the effective stress. The equations for effective stress are repeated below for convenience.

Miner's method

$$S_e = \left[\frac{\sum n_i S_i^b}{\sum n_i} \right]^{1/b} \tag{2-8}$$

Corten-Dolan method

$$S_e = \left[\frac{\sum n_i S_i^d}{\sum n_i} \right]^{1/d} \tag{2-23}$$

Shanley's 2x method

$$S_e = \left[\frac{\sum n_i S_i}{\sum n_i} \right]^{1/2b} \tag{2-35}$$

The Shanley theory yields the most conservative result. The corresponding time at the effective stress is

$$t_e = \frac{\sum n_i}{f}. \tag{3-18}$$

Random Excitation. Frequently a sinusoidal dwell, or multiple sinusoidal dwells, occuring at different frequencies will not duplicate a field condition with suitable accuracy. In this case a random vibration loading is used to achieve a more realistic test. Once an original random spectrum is defined or assumed, an equivalent random test may be desired which is modified in level to achieve a change in test time. The environmental engineer may also wish to convert a series of field experiences, which vary only in energy level, to a single-level equivalent spectrum. The time scaling of a stationary random-vibration test is handled in much the same manner as the time scaling of the single-frequency sinusoidal-dwell test. In this case the rms stress level is defined by

$$\ddot{S} = C_r [f_n Q W(f_i)]^{0.5}, \tag{3-19}$$

where $W(f_i)$ represents the excitation spectral density in the bandwidth of specimen resonance.

In the preceding development of random-excitation time-scale equivalence it was assumed that the random process was stationary, that is, the energy distribution vs frequency envelope was constant with time. It was also assumed that the excitation and response were Gaussian processes. As noted by Smits [56], a non-Gaussian random-loading history or test input are poorly understood stochastic processes. Narrowband nongaussian processes may be approximately characterized by features such as the average frequency of mean level crossings and crest distribution. More verified research is required before reliable models may be selected for equivalence purposes. The use of the Gaussian probability density function is defended by Poppleton [37] and others, who note that many important structural inputs appear to be good approximations of a Gaussian process. Further, there is a large body of literature concerned with the Gaussian distribution, and a Gaussian random-noise generator is a convenient excitation source for the electromagnetic-vibration testing machine.

Time-Scaling Practices

Table 3-1 is provided as a summary of the vibration equivalence time-scaling techniques developed in Section 3.2.

The assumption that a single energy level and time scale change which will produce an equivalence test simultaneously at each individual response frequency, for a specimen which responds at several frequencies, is usually not valid. As a result it is necessary to identify all potentially critical sections of the specimen. A test is then designed which will produce equivalence for one or more critical sections, and which will undertest the remaining sections. For a linear specimen with no damping,

$$W(f_i)_2 = W(f_i)_1 t_R^{1/b} \tag{3-20}$$

or

$$\hat{W}(f_i)_2 = \hat{W}(f_i)_1 t_R^{1/b}, \tag{3-21}$$

where $W(f_i)$ and $\hat{W}(f_i)$ are the excitation and response spectral densities, respectively.

Table 3-1. Test Time-Scaling Practices

Original Spectrum	Final Spectrum			Notes
	Undamped Linear	Damped Linear	Nonlinear	
1.0 Sinusoidal Dwell †1.1 Single load amplitude, single frequency	$\frac{V_2}{V_1} = \frac{\hat{V}_2}{\hat{V}_1} = t_r^{1/b}$	$V_2 = V_1 t_R^{(n-1)/b}$	Same as damped or undamped, except $\hat{t}_2 = t_2 \frac{f_1}{f_2}$	b characterizes the S-N curve corrected for temperature, configuration, and stress ratio
1.2 Multiple load amplitudes, single frequency	a) Each load amplitude scaled separately, as in 1.1, and applied in same sequence as original excitation, or b) Compute an equivalent stress (see Eqs. (2-8), (2-23), and (2-35)) and equivalent time (Eq. (3-18)). Then scale as in 1.1 as applicable.			See 1.1
1.3 Single or multiple load amplitudes at different frequencies	Scale each frequency independently using 1.1 and 1.2 as applicable.			See 1.1 and 1.2
2.0 Sinusoidal Sweep	Scale as in 1.1 or 1.2 except $t_R = \frac{\text{original no. sweeps}}{\text{final no. sweeps}}$			See 1.1 and 1.2
3.0 Random Excitation 3.1 Single envelope	$\frac{W(f_i)_2}{W(f_i)_1} = \frac{\hat{W}(f_i)_2}{\hat{W}(f_i)_1} = t_R^{1/b}$	$W(f_i)_2 = W(f_i)_1 t_R^{n/b}$	Correction of test time based on most critical response frequency $\hat{t}_2 = t_2 \frac{f_1}{f_2}$	In this case b characterizes an appropriate $\bar{S}$-N curve. The random signal is stationary and Gaussian.
3.2 Multiple envelope	Scale each envelope individually per 3.1 and apply sequentially			See 3.1

†See Eq. (3-15) for definition of t_R.

Damping is accommodated in a manner similar to that which was employed for the single-frequency sinusoidal dwell, except that Eq. (3-19) is used to define the stress level that will result from $W(f_i)$. As a result, for the linear system,

$$W(f_i)_2 = W(f_i)_1 t_R^{n/b}. \tag{3-22}$$

Should the specimen exhibit a nonlinear response at the frequencies which elicit the critical stresses, the scaled test duration should be corrected as described by Eq. (3-14).

3.3 Change-of-Test-Type Equivalence

Preliminary Comment

In addition to developing and using equivalence procedures to design tests which have been time scaled for convenience in laboratory testing, the fatigue-based equivalences have been used to equate various test types for comparison purposes. Change-of-test-type equivalences are also important to the product designer. They enable him to predict how his design will react in an application involving a new dynamic environment if he knows how the design responded to some other dynamic environment.

There are a few general observations which may be made relative to the change-of-test-type equivalences: (a) there is a need to have these equivalences, (b) various theories have been developed to provide sets of equivalence relationships, and (c) there are very little reliable data available which may be cited to substantiate the equivalences. The lack of proven substantiation is unfortunate; however, the need for change-of-test-type equivalences is persistent and this need is expected to stimulate a continued search for reliable change-of-test-type techniques.

The remainder of this section covers basic work in change-of-test-type equivalences and ends with a tabular summary of the resultant equivalence relationships.

Miles

The change-of-vibration-test-type equivalence was probably first considered by Miles [57] in 1954. Miles noted that there are three basic subsets of the general problem of determining the stresses in a specimen and the consequent possibility of fatigue failure: (a) the statistical description of the original loading, (b) the statistical description of the dynamic response of the specimen to the applied loading, and (c) determination of the probability of fatigue failure associated with the response.

He assumed that the specimen was an elastic structure, that it could be represented by a lightly damped SDF oscillator, and that the specimen response has sharp frequency selectivity associated with resonance. The frequency response characteristics of the narrowband oscillator are illustrated in Fig. 3-8.

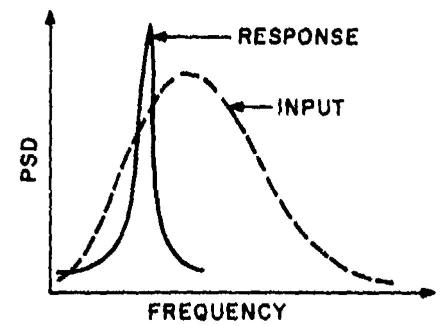


Fig. 3-8. Response of a narrow-bandwidth oscillator to a wideband input.

Both the linear cumulative-damage hypothesis of Miner and the nonlinear hypothesis of Shanley were considered. The Miner hypothesis was selected because Miles concluded that the Shanley theory leads to insignificant changes in life relative to the Miner theory when stress amplitudes are distributed continuously over a wide range. The Miles approach involved the selection of a single sinusoidal stress level which would produce the same fatigue damage as a random load spectrum. The resultant equivalent stress equation is

$$S_e = \left[\frac{\sum n_i S_i^{k\alpha}}{\sum n_i}\right]^{1/(k\alpha)} , \qquad (3\text{-}23)$$

where the function $k\alpha$ is a constant determined by test or derived from an assumed mathematical model of the stress field. The assumed mathematical model may or may not make reference to a material S-N curve. If $k = 1$ and $\alpha = b$ or d, the expression would equal those of Miner or Corten-Dolan, respectively. The Shanley theory is duplicated when $k = 2$ and $\alpha = b$. The range of k is imprecisely defined but probably does not exceed 2. Equation (3-23) was applied by Miles. He assumed a narrowband continuous stress spectrum to provide an expression for the probable equivalent stress,

$$S_e = \left[\frac{\int_0^\infty S^{k\alpha} P(S)}{\int_0^\infty P(S)ds}\right]^{1/(k\alpha)} , \qquad (3\text{-}24)$$

which was integrated for a lightly damped SDF oscillator (as shown in Fig. 3-9) to yield

$$S_e = (2S^{-2})^{1/2}\left[\Gamma\left(\frac{k\alpha}{2} + 1\right)\right]^{1/(k\alpha)}. \qquad (3\text{-}25)$$

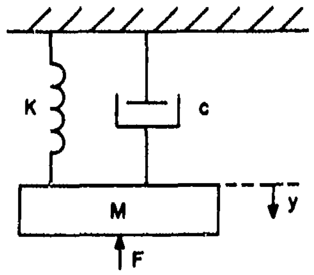


Fig. 3-9. An SDF oscillator.

And, assuming the Rayleigh distribution for $P(S)$,

$$P(S) = 0.5S \exp\left(\frac{-S^2}{2\overline{S}^{-2}}\right) . \tag{3-26}$$

Miles then simplified Eq. (3-25) by the use of Stirling's formula to approximate the gamma function,

$$S_e \approx (\pi k\alpha)^{1/(2k)}\left(\frac{k\alpha\overline{S}^2}{e}\right)^{1/2} \approx \left(\frac{k\alpha\overline{S}^2}{e}\right)^{1/2}, \tag{3-27}$$

where e is the base of Naperian logarithms and small terms are neglected.

The preceding analysis led to the central result of Miles' work where he related an equivalent stress to the stress produced by a static loading of the resonant structure with an rms force $\overline{F}$,

$$\frac{S_e}{S_0} = \left(\frac{k\alpha\pi}{4e\delta}\ \frac{\omega_0 W[F]}{\overline{F}^2}\right)^{0.5}, \tag{3-28}$$

where

ω_0 = the resonant frequency of the structure
S_0 = the stress resulting from a static load application of the rms force $\overline{F}$
δ = the ratio of structural damping to critical damping
$W[F]$ = the power spectral density (PSD) of the force in the neighborhood of resonance.

This relationship was applied to two cases. The first was the case of a monotonically decreasing load (i.e., $W[F_d]$ decreases as frequency increases) of the form

$$W[F_d] = \left(\frac{2\overline{F}^2}{\omega_1\sqrt{\pi}}\right)\exp\left[-\left(\frac{\omega}{\omega_1}\right)^2\right], \tag{3-29}$$

where subscript d refers to the decreasing load spectrum, and ω_1 is the characteristic frequency usually selected as the frequency at the half-power point. The second case was a load spectrum of the Gaussian form which exhibits a peak at some angular frequency ω_m, as given by

$$W[F_p] = \left(\frac{4\overline{F}^2}{\omega_m\sqrt{\pi}}\right)\left(\frac{\omega}{\omega_m}\right)\ \exp\left[-\left(\frac{\omega}{\omega_m}\right)^2\right] , \tag{3-30}$$

where the subscript p refers to a peaked or Gaussian spectrum.

Of the preceding expressions for the monotonically decreasing load and the peaked or Gaussian spectrum, probably the peaked spectrum is the most representative random load distribution encountered in service. The corresponding stress ratio for the monotonic load is

$$\frac{S_e}{S_0} = \left(\frac{\sqrt{\pi}}{2e}\right)^{1/2}\left(\frac{k\alpha}{\delta}\right)^{1/2}\left(\frac{\omega_0}{\omega_1}\right)^{1/2} \exp\left[-\frac{1}{2}\left(\frac{\omega_0}{\omega_1}\right)^2\right], \tag{3-31}$$

and the stress ratio for the peaked spectrum load is

$$\frac{S_e}{S_0} = \left(\frac{\sqrt{\pi}}{e}\right)^{1/2}\left(\frac{k\alpha}{\delta}\right)^{1/2}\left(\frac{\omega_0}{\omega_1}\right)^{3/2} \exp\left[-\frac{1}{2}\left(\frac{\omega_0}{\omega_1}\right)^2\right]. \tag{3-32}$$

The corresponding upper bound on stress ratio for the peaked spectrum was computed by Miles to be

$$\left.\frac{S_e}{S_0}\right]_{\max} \approx 0.51\left[\frac{k\alpha}{\delta}\right]^{1/2}. \tag{3-33}$$

The use of typical values ($\alpha = 10$, $k = 2$, and $\delta = 0.02$) yields $S_e/S_0 = 16$.

Spence and Luhrs

Spence and Luhrs [58,60] extended the Miles analysis by developing a method by which different random or combined random plus sinusoidal vibration excitations may be compared. In the Spence and Luhrs derivation, substitute sinusoidal vibration excitations were developed based on the response characteristics of a single resonator system, and amplitude and time relationships were based on the usual appropriately selected S-N diagrams.

The Spence and Luhrs analysis utilized an expression for the amplitude probability distribution of a combined constant-amplitude sine wave and Gaussian signal with Eq. (3-24). The result was an expression for the equivalent sinusoidal stress in terms of a combined rms sinusoidal and random stress

$$\frac{S_e}{S_c} = \frac{2}{(2+r^2)^{1/2}}\left[\left(\frac{k\alpha}{2}!\right)^2 \sum_{i=0}^{k\alpha/2} \frac{\left(\frac{1}{2}r^2\right)^i}{(i!)^2\left(\frac{1}{2}k\alpha - i\right)!}\right]^{1/(k\alpha)}, \tag{3-34}$$

where

S_c = the combined sine and random stress of the form

$$S_c^2 = \bar{S}_s^2 + \bar{S}_r^2 ; \tag{3-35}$$

$\bar{S}_s$ = rms sinusoidal stress

$\bar{S}_r$ = rms random stress

r = the ratio of peak sine to rms random stress

i = the summing integer.

Equation (3-34) was evaluated by the following manipulations. First, it was noted that $k\alpha/2$ was restricted to integer values during development of the equivalent stress equation. Second, the weighting factor k was considered to range between the value of 1 and the conservative Shanley limit of 2 [27]. Third, the relationship between α and the slope of the linear portion of an S-N diagram was defined as

$$\alpha = b \tag{3-36}$$

in accordance with the definition of b per Fig. 2-6. Fourth, the fatigue relationship of Eq. (3-34) was related to an actual dynamic system by assuming that the response of a single resonator would constitute a suitable mathematical model for fatigue failure in an actual specimen,

$$V_{sd,e} = \frac{S_e}{S_c}\left(\frac{\pi}{2}\,\frac{W[V]f_n}{Q} + \bar{V}^2\right)^{0.5}, \tag{3-37}$$

where

$V_{sd,e}$ = sine dwell substitute excitation, g peak
$W[V]$ = PSD of the excitation in g^2/Hz
f_n = the natural frequency of the oscillator in hertz
Q = the resonant transmissibility of the oscillator
$\bar{V}$ = rms sine excitation, grams.

Fifth, Eq. (3-37) was modified to achieve the form,

$$\bar{V}_{sd,e} = \bar{V}\frac{S_e}{S_c}\left(\frac{1}{A^2} + \frac{1}{2}\right)^{0.5}, \tag{3-38}$$

where $\bar{V}_{sd,e}$ is the rms sinusoidal substitute equivalent excitation; and finally,

$$A = \frac{V}{\left(\dfrac{\frac{\pi}{2}W[V]f_n}{Q}\right)^{0.5}} = \frac{2\bar{V}}{\left(\dfrac{\pi W[V]f_n}{Q}\right)^{0.5}}\,. \tag{3-39}$$

The variable A was plotted in Fig. 3-10.

It was assumed that $W[V]$ was a constant value (i.e., white) over the entire frequency range of interest. This assumption was considered to introduce little error because only the value of $W[V]$ in the vicinity of f_n would be expected to

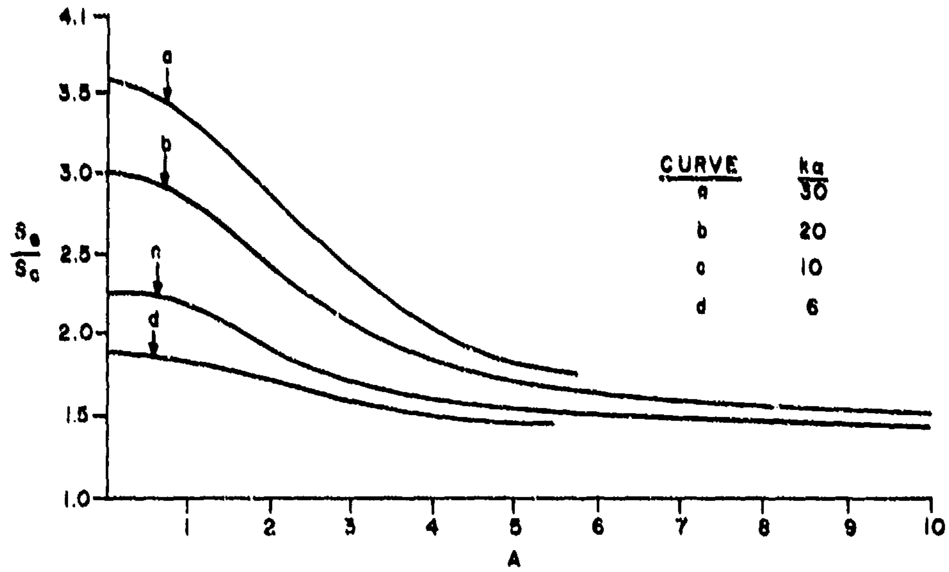


Fig. 3-10. Ratio of S_e/S_c vs A for various values of $k\alpha$ [59].

cause significant specimen response. The only restriction of $W[V]$ is that it is flat in the vicinity of f_n. Kaufman et al. [61] may be consulted for the case where $W[V]$ varies greatly in the vicinity of f_n.

When the desired substitute excitation is a fixed-frequency sinusoidal signal, which is intended to represent a random excitation combined with swept sinusoidal excitation, the formation of an equivalence consists of two steps. The first step is to account for resonator response when the swept sinusoidal component passes through f_n. The second step is to account for resonator response when the swept sinusoidal component is away from resonance and the resonator is excited only by the random components at f_n.

Two assumptions are necessary at this point to enable computation of an equivalent time at resonance for sinusoidal signal swept through f_n. First, amplitude buildup of the resonator response is independent of sweep rate, which is true only for relatively slow sweep rates (recall Eq. (3-16)). Second, the time at resonance is considered to be the time required for the sweeping signal to move between half-power points of the resonator [62]. For a constant sweep rate the time at resonance t_c is

$$t_c = \frac{60 f_n}{Q \cdot (\text{rate})}, \tag{3-40}$$

which was plotted as Fig. 3-11. For a logarithmic sweep rate the time at resonance t_l is

$$t_1 = \left[\frac{60 \ln\left(\frac{Q + 0.5}{Q - 0.5}\right)}{(\text{rate})\ln 2}\right], \tag{3-41}$$

which was plotted as Fig. 3-12.

When the sinusoidal component is away from the resonance band and only the random components are present at resonance, the sinusoidal substitute value [57,58] is

$$\overline{V}^{r}_{sd,e} = \frac{S_e}{S_c}\,\frac{\overline{V}}{A}, \tag{3-42}$$

where the superscript denotes the original signal type, the subscript denotes the new signal type, and e symbolizes equivalence. The time duration $t^{r}_{sd,e}$ associated with the rms random equivalent sinusoidal test signal $\overline{V}^{r}_{sd,e}$ is the total time duration of the original envelope minus the time consumed sweeping through resonance. For a constant sweep rate,

$$t^{r}_{sd,e} = t_t - t_c, \tag{3-43}$$

where t_t is the total time duration of the original signal and t_c is the time at constant sweep rate when the excitation signal is sweeping through resonance. For a logarithmic sweep rate,

$$t^{r}_{sd,e} = t_t - t_1, \tag{3-44}$$

where t_1 is the time during a logarithmic sweep rate when the excitation signal is sweeping through resonance.

The two components, an equivalent rms sinusoidal signal simulating a swept-frequency component and an equivalent rms sinusoidal signal simulating a random component, are adjusted in level and time duration in a manner similar to that used for test time scaling. That is,

$$\frac{\overline{V}^{ss}_{sd,e}}{\overline{V}^{r}_{sd,e}} = \left(\frac{N^{r}_{sd,e}}{N^{ss}_{sd,e}}\right)^{1/b} \tag{3-45}$$

where

$N^{r}_{sd,e}$ = the number of cycles to failure at the equivalent rms sinusoidal dwell level which represents an original random excitation

$N^{ss}_{sd,e}$ = the number of cycles to failure at the equivalent rms sinusoidal dwell level which represents an original swept sinusoidal excitation.

The preceding is accurate only if stress is directly proportional to acceleration. If stress is a nonlinear function of acceleration (or loading) then suitable modifications are required to accurately apply Eq. (3-45).

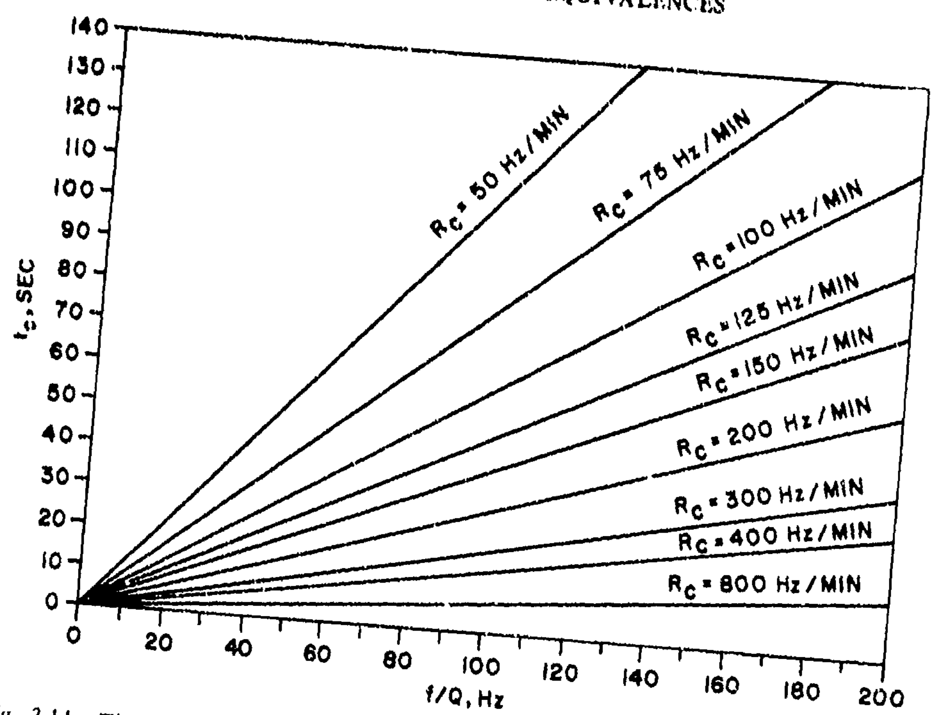


Fig. 3-11. Time spent in passing through a resonance using a constant sweep rate [59].

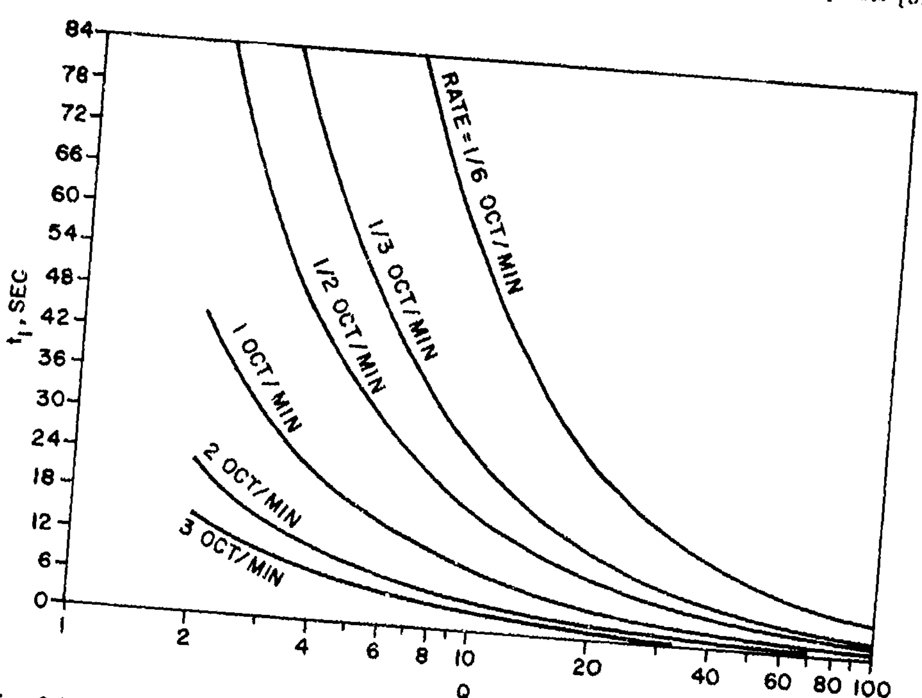


Fig. 3-12. Time spent in passing through a resonance using a logarithmic sweep rate [59].

The substitute sinusoidal dwell signal, as derived by Spence and Luhrs from the work of Miles, is applied using the specimen's natural frequency, the derived equivalence level, and the appropriate time of application. Table 3-2 is presented to summarize and organize the computations required to find an rms sine substitute excitation for either a combined random plus swept sine or a random only signal.

Table 3-2. The Spence and Luhrs Computational Sequence

Step	*Comment*	*Using*
1.0 Define resonator model	f_n, Q, and b	Measurements, analysis, or intuition
2.0 Define original excitation	Time of exposure, $W[V]$ as a function of frequency, and as appropriate, superimposed sine sweep or dwell level, sweep rate, and frequencies	
3.0 Find S_e/S_c		
3.1 Determine A		Eq. (3-39)
3.2 Find S_e/S_c	Select appropriate b and assume $1 \leqslant k \leqslant 2$. If no sinusoidal component, then read S_e/S_c at $a = 0$.	Fig. 3-3
4.0 Compute $\overline{V}_{sd,e}$	Equivalent rms sine level for combined random and swept sine which acts for t_1 or t_c	Eq. (3-38)
5.0 Compute $\overline{V}^r_{sd,e}$	Equivalent rms sine level for random only	Eq. (3-42)
6.0 Define test time		
6.1 For combined random and sine	Depends on type of sweep (logarithmic or constant rate)	(Eq. (3-40) or (3-41)
6.2 For random only	Depends on type of sweep (logarithmic or constant rate)	Eq. (3-43) or (3-44)
6.3 Normalize	Adjust $\overline{V}_{sd,e}$ to same basis, either 4.0 or 5.0	Eq. (3-45)
6.4 Total	Equivalent test time at f_n for the normalized level of 6.3	Sum of 6.1 and 6.2

Crede and Lunney

Crede and Lunney [63,64] conducted extensive studies for establishing enlightened shock and vibration test requirements for missile and airborne electronic equipment. They considered the nature of equipment responses and found these responses to be more important, in the evaluation of test severity, than the characteristics of the input vibration. This conclusion followed an observation that the stress level was more directly related to the responses of equipment structures rather than to an input excitation.

The authors advocated the use of a two-level test, based on curves which envelop appropriate sets of field data measured at the equipment mounting points. An operational test would be performed using the measured vibration as the test input level. A nonoperational or structural endurance test would be performed by exciting the equipment at an increased input level for the purposes of reducing rest time. The accelerated test was derived using *S-N* curve information as discussed in Section 3.2.

In the case of airborne electronics, the excitation vibration was considered to be solely sinusoidal. In the case of missile electronics, excitation vibration was considered to be represented by a continuous spectrum.

The suggested equivalent test consisted of a substitute sinusoidal test. A Miles type of analysis was used to derive equivalent stresses for two cases:

- Continuous spectrum excitation
- Beat excitation.

The continuous or random spectrum involved the Miles analysis approach to derive an equivalent stress relationship. Equation (3-27) is repeated here for convenience:

$$S_e \approx \left(\frac{k\alpha\bar{S}^2}{e}\right)^{0.5}. \qquad (3\text{-}27)$$

The Miles expression was further simplified by assuming that $k = 2$ and $\alpha = 10$ were representative parameters, with the result that

$$S_e \approx 2.71\bar{S}. \qquad (3\text{-}46)$$

The choice of parameters k and α must be carefully made. For example, it was noted earlier in the discussion of Miles' work that $k = 1$ and $\alpha = 10$ were typical values. Thus from Eq. (3-27),

$$S_e \approx 1.92\bar{S}, \qquad (3\text{-}47)$$

which is more in line with data from Granick [65] and McIntosh and Granick [66].

A beat excitation signal was considered to be the sum of two or more discrete spectra. The discrete spectra may result from an interposition of vehicle structure between the primary excitation source (aerodynamic buffeting, engine, other) and the equipment mounting location. Because structures tend to respond at specific resonance frequencies when loaded by a continuous or nearly continuous excitation, it is these resonance "bandpass" frequencies which eventually combine to form the excitation spectrum for an individual item of equipment in or on the vehicle. The resultant discontinuous excitation spectrum may consist of periodic beats of varying amplitude, or it may consist of aperiodic beats. Crede and Lunney considered the case of periodic beats resulting from two equal-amplitude steady state components which were slightly different in frequency. The resultant beat excitation was assumed to periodically reduce to zero. The resultant beat excitation is shown in Fig. 3-13.

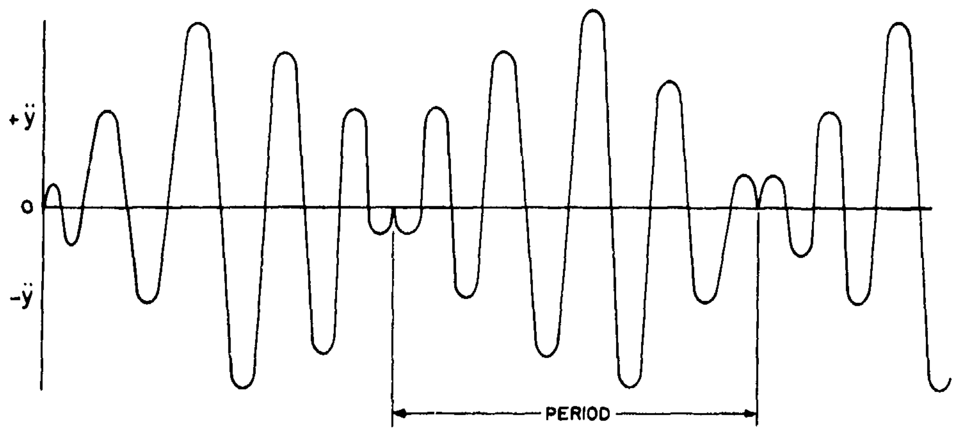


Fig. 3-13. Beat excitation (for two nonharmonically related frequencies).

The equivalence problem was resolved by finding an effective vibration level which would provide the same damage as the fluctuating stress. The equivalent stress was given in terms of the peak stress S_{sp} caused by the beat excitation;

$$S_e = \frac{1}{3}[S_{sp} + (12\bar{S}^2 - 2S_{sp}^2)^{1/2}], \tag{3-48}$$

with the result

$$S_e \approx 1.3\bar{S},$$

where $S_{sp}/\bar{S} = 2.1$.

The difference between continuous and beat spectra is due to differences in the relative percentage of low-amplitude stresses. Comparison of Eq. (3-49) with

Eq. (3-46) illustrates the necessity of knowing the appropriate probability distribution functions for the response stresses in order to select proper equivalent test levels.

Mains

Mains [67] attempted a generalized approach to the problem of damage accumulation in environmental testing. Although his analysis relates to fatigue damage by use of Miner's theory, the approach could be applied to other types of damage. The degree of damage was assumed to be equal to the product of three factors,

$$D = A_1 A_2 A_3, \tag{3-50}$$

where

A_1 = an amplitude of maximum response factor
A_2 = a distribution of load amplitudes factor
A_3 = a material and structural factor.

There are four assumptions which are basic to the Mains approach.

1. A damped-oscillator model similar to that of Miles (Fig. 3-9) will characterize the specimen and respond to a transient load in the usual manner,

$$y = \exp(-\delta \omega_n t) C_2 \sin \omega_n t + y_0(t), \tag{3-51}$$

where

y = displacement or stress or strain
ω_n = natural response frequency.

2. A linear approximation suitably describes the logarithmic S-N curve in the region of interest,

$$yN^k = C, \tag{3-52}$$

where k, y, and C for our case of fatigue damage are analogous to b, S, and C_1 from Eq. (2-3)

$$yN^k \approx SN^{1/b} \approx C^{1/b}. \tag{3-53}$$

3. If yielding is involved then the assumed amplitude decay (i.e., the exp $(-\delta\omega_n t)$ term of Eq. (3-51)) is too slow and must be adjusted based on experiment.

4. Damage accumulation is linear in accordance with Miner's theory (recall Eq. (2-5).

With this basis established, the damage is expressed as

$$D = \left(\frac{C_{max}^b}{C}\right)\sum\left(\frac{C_{2m}}{C_{max}}\right)^b\left\{\left[1 + \exp\left(\frac{-\pi\delta}{\sqrt{1-\delta^2}}\right)\right]^b \times \right.$$

$$\left.\sum_r^{1/\delta} \exp\left(\frac{-\delta\pi b}{2} \cdot \frac{4\bar{n}-3}{\sqrt{1-\delta^2}}\right)\right\}, \tag{3-54}$$

where

C_{max} = the largest initial response in the series of transients
C_{2m} = the initial response of the different (mth) transient
$\bar{n}$ = the number of cycles in the transient.

Assuming that the previous work was correct, Mains presented a series of relationships between relative damage and various individual parameters. These relationships are given in Table 3-3. Although the approach taken by Mains is not supported by data, it or a similar approach could serve as a basis for further research on damage accumulation.

Table 3-3. Relative Damage as Affected by Various Factors†

Factors	*Relationship* ‡
1.0 Change of test amplitude	$\frac{D_1}{D_2} = \left(\frac{C_{max\ 1}}{C_{max\ 2}}\right)^{1/k}$
2.0 Change of n at constant load amplitude	$\frac{D_1}{D_2} = \frac{N_1}{N_2}$
3.0 Change in test time only at constant cycle rate and load	$\frac{D_1}{D_2} = \frac{t_1}{t_2}$
4.0 Change in damping ratio	$\frac{D_1}{D_2} = \frac{(A_3)_1}{(A_3)_2}$
5.0 Change in strength of material	$\frac{D_1}{D_2} = \left(\frac{C_2}{C_1}\right)^{1/k}$
6.0 Change in strength of material and slope of log S-vs-log N curve	$\frac{D_1}{D_2} = \frac{C_2^{1/k}\,2(A_2)_1(A_3)_1}{C_1^{1/k}\,1(A_2)_2(A_3)_2}$

† From Mains [67].
‡ The subscript 1 refers to the new or changed condition; subscript 2 refers to a reference condition; k and C are defined in Eq. (3-52); and A_1, A_2, and A_3 come from Eq. (3-50).

Hall and Waterman

Hall and Waterman [68] provided one of the earlier attempts to develop a complete set of equivalence relationships between sinusoidal and random vibration histories. Their work is documented here for reference purposes, although it has not found wide application in recent years. The basis for equivalence was the amount of work expended due to damping forces, and it was assumed that each type of vibration history would produce the same amount of damage on a second-order system model. The equivalence expressions were derived for sinusoidal dwell at resonance, sinusoidal sweep, and random vibration.

The work expressions from the original paper are given below.

Sinusoidal dwell at resonance

$$W_{sd,i} = \frac{\pi n_i \bar{m}^2 V_{sd,i}^2 Q g^2}{M_1 \omega_n^2} , \tag{3-55}$$

where W refers to work.

Random

$$W_{r,i} = \frac{t_{r,i} g^2 \bar{m}^2 W[V]}{4M_1} . \tag{3-56}$$

Sinusoidal sweep at constant rate

$$W_{ss,i} = \frac{\bar{m}^2 V_{ss,i}^2 g^2}{8(\text{rate})M_1} , \tag{3-57}$$

where

W = work
i = a subscript identifying a specific load level
$\bar{m} = \Sigma M_i f_i$, a generalized mass
$M_1 = \Sigma M_i f_i^2$, a simplifying substitution
T_i = the total time at load level i
(rate) = the constant sweep rate in Hz/sec.

Equivalence was established from the work expressions by equating work terms. The resultant relationships were given in pairs.

Sinusoidal dwell–random

$$\left.\begin{aligned} V_{sd,e}^{r} &= \left(\frac{W[V]\omega_n}{2Q}\right)^{0.5} \\ W[V]_e^{sd} &= \frac{2QV_{sd,i}^2}{\omega_n} \end{aligned}\right\} \tag{3-58}$$

Sinusoidal sweep–random

$$\left.\begin{aligned} V^{r}_{ss,e} &= \left[\left(\frac{t_i}{t_0}\right) 2W[V]\Delta f_0\right]^{0.5} \\ W[V]^{ss}_{e} &= \left(\frac{t_i}{t_0}\right)\frac{V^2_{ss,i}}{2\Delta f_0} \end{aligned}\right\} \tag{3-59}$$

where

f_0 = an octave bandwidth

t_0 = time to sweep one octave.

The sinusoidal dwell–sinusoidal sweep pair was not presented; however, by noting that

$$t_{sd,i} = \frac{n_i}{\omega_n}, \tag{3-60}$$

the last pair is easily derived:

Sinusoidal sweep–sinusoidal dwell

$$\left.\begin{aligned} V^{ss}_{sd,e} &= V_{ss,i}\left[\left(\frac{t_0}{t_{sd,i}}\right)\frac{\omega_n}{8\pi Q\Delta f_0}\right]^{0.5} \\ V^{sd}_{ss,e} &= V_{sd,i}\left[\left(\frac{t_{sd,i}}{t_0}\right)\frac{8\pi Q\Delta f_0}{\omega_n}\right]^{0.5} \end{aligned}\right\} \tag{3-61}$$

Waterman [69] extended the primary analysis and noted that the equivalence philosophy was that damage is cumulative and linearly related to time (cycles) of load application at a given load level. Waterman assumed that

$$D = \ddot{S}^{\alpha} W, \tag{3-62}$$

where damage is equal to the product of work due to damping and rms response raised to the α power. The parameter α is similar to the S-N curve parameter b. The difference is that α characterizes the slope of an amplitude-vs-time plot constructed by testing a model to failure at several load levels. This approach provides an equivalence based more on engineering philosophy rather than mathematical rigor. Waterman proposes $\alpha = 2$ for steel and $\alpha = 0$ for aluminum based on limited experimental test data. These values should be verified by more data prior to use.

The resultant damage expressions are given below.

Sinusoidal dwell

$$D_{sd,i} = \frac{(Q^{\alpha+1})(g^{\alpha+2})(V_{sd,i}^{\alpha+2})\bar{m}t_{sd,i}}{(2^{(\alpha+2)/2})(\omega_n^{2\alpha+1})}, \tag{3-63}$$

which was not derived by Waterman but was included for completeness.

Random

$$D_{r,i} = \frac{\bar{m}(g^{\alpha+2})(W[V]^{(\alpha+2)/2})(Q^{\alpha/2})t_{r,i}}{(2^{\alpha+2})(\omega_n^{3\alpha/2})}. \tag{3.64}$$

Sinusoidal sweep

$$D_{ss,i} = \frac{(1.443\pi)^{(\alpha+2)/2}(g^{\alpha+2})(Q^{\alpha+2})(V_{ss,i}^{\alpha+2})\bar{m}t_{ss,i}}{(2^{\alpha+2})(\omega_n^{2\alpha+1})(n_0^{(\alpha+2)/2})}, \tag{3-65}$$

where n_0 = the total number of octaves swept.

Equivalences between various types of vibration were obtained by equating damage functions, and the relationships again appear in pairs.

Sinusoidal swell–random

$$\begin{aligned} V_{sd,e}^{r} &= \left[\frac{t_{r,i}}{t_{sd,e}}\right]^{1/(\alpha+2)}\left[\frac{W[V]\,\omega_n}{2Q}\right]^{1/2} \\ W[V]_e^{sd} &= \left[\frac{t_{sd,i}}{t_{r,e}}\right]^{2/(\alpha+2)}\left[\frac{2Q\,V_{sd,i}^2}{\omega_n}\right] \end{aligned} \tag{3-66}$$

Sinusoidal sweep–random

$$\begin{aligned} V_{ss,e}^{r} &= \left[\frac{t_{r,i}}{t_{ss,e}}\right]^{1/(\alpha+2)}\left[\frac{N_0\omega_n W[V]}{1.443\pi}\right]^{1/2} \\ W[V]_e^{ss} &= \left[\frac{t_{ss,i}}{t_{r,e}}\right]^{2/(\alpha+2)}\left[\frac{1.443\pi\,V_{ss,i}^2}{n_0\omega_n}\right] \end{aligned} \tag{3-67}$$

Sinusoidal sweep–sinusoidal dwell

$$\begin{aligned} V_{sd,e}^{ss} &= V_{ss,i}\left[\frac{t_{ss,i}}{t_{sd,e}}\right]^{1/(\alpha+2)}\left[\frac{0.722\pi}{n_0 Q}\right]^{1/2} \\ V_{ss,e}^{sd} &= V_{sd,i}\left[\frac{t_{sd,i}}{t_{ss,e}}\right]^{1/(\alpha+2)}\left[\frac{n_0 Q}{0.722\pi}\right]^{1/2}. \end{aligned} \tag{3-68}$$

Equations (3-66), (3-67), and (3-68) show that when test times are equal, the test amplitudes are independent of α.

Gerks

Gerks [26,70] presented a complete set of equivalence relationships between sine sweep, sine dwell, and random vibration histories. Both the Miner and the Corten-Dolan fatigue-damage accumulation theories were applied in a Miles type of analysis to form equivalence expressions. The resulting equivalence equations recognize the dynamic characteristics of specimen natural frequency and transmissibility. A linear SDF system was used as an idealized dynamic model; however, the analysis is applicable to higher mode responses. It was assumed that any complex equipment may be represented by a combination of noninteracting SDF oscillators. It was also assumed that the total damage caused by a combination of vibration spectra was the sum of the piecewise damage caused by each individual spectrum. The case of nonstationary or time-varying vibration was treated by dividing the environment into a series of sequential environments, each with a set of constant parameters.

The resulting expressions equate fatigue damage in terms of vibration input parameters rather than stress or strain. Gerks indirectly validated the expressions by conducting a limited series of tests on aluminum alloy cantilever beams with end masses and viscoelastic damping. As a result of these tests, the expressions based on the Miner fatigue hypothesis were found to be more accurate than those based on the Corten-Dolan theory. The following discussion will cover the general assumptions required to generate the equivalence expressions; however, only those relationships based on Miner's theory are presented.

The following assumptions were made to reduce the complexity of deriving the equivalence expressions:

1. The logarithmic *S-N* curve is linear in the range of interest.
2. The various types of vibration occur sequentially.
3. More than one vibration-excitation component could exist at one time; however, these components would have sufficient frequency separation to avoid simultaneous excitation of any one resonance.
4. The vibration-excitation amplitude is constant and varies insignificantly in the region of each specimen resonance.
5. The field vibration is stationary or can be represented as a series of sequential stationary environments.
6. Sine sweep rates were assumed to be sufficiently slow so that the resulting specimen response to the sweep excitation could be approximated by a dwell response.

The application of the Gerks equivalence expressions is a four-step procedure:

1. Choose the final test duration and type of equivalent test which is desired, i.e., sine dwell, sine sweep, or random.

2. Combine all like types of vibration history into one test of each type lasting for the same duration as chosen in Step 1. The result would be, as applicable; all sine sweeps into one equivalent sine sweep, all sine dwells into one equivalent sine dwell, and all random envelopes into one equivalent random envelope.

3. Transform each equivalent test type into the preselected or chosen type of test.

4. Combine the like types of vibration from Step 3 into the final equivalent test level.

A diagram of the reduction procedure as outlined above is presented in Fig. 3-14.

Three types of expressions were developed: First, expressions which are used when each type of vibration history must be combined into one history for some desired exposure time. Second, expressions used when each combined history must be transformed into one final type of history. Last, expressions used when each transformed history must be combined into one final equivalent test.

The expressions for combining like vibration histories into one history of the same type are as follows:

Several sine dwells to one sine dwell

$$V_{sd,e} = \left[\sum_{i=1}^{m} \sum_{j=1}^{p} \frac{f_{i,j}}{f_n} \frac{t_i}{t_e} \frac{(V_{sd})_{i,j}^{b}}{\left\{ Q^2 \left[1 + \left(\frac{f_{i,j}}{f_n} \right)^4 \right] - \left(\frac{f_{i,j}}{f_n} \right)^2 (2Q^2 - 1) \right\}^{b/2}} \right]^{1/b} \tag{3-69}$$

Step 1	*Original Vibration*	*Step 2*	*Step 3*	*Step 4*
Choose final test type and duration (assume sine sweep for time t_e)	$\sum_i V_{sd,i}, t_{sd,i} \rightarrow$	$V_{sd,e}^{sd}, t_e \rightarrow$	$V_{ss,e}^{sd}, t_e$	
	$\sum_i V_{ss,i}, t_{ss,i} \rightarrow$	$V_{ss,e}^{ss}, t_e$	$\longrightarrow$	$\rightarrow V_{ss,e}, t_e$
	$\sum_i W[V]_i, t_{r,i} \rightarrow$	$W[V]_e^r, t_e \rightarrow$	$V_{ss,e}^r, t_e$	

Fig. 3-14. Equivalent test reduction procedure (an example) [26].

Several logarithmic sine sweeps to one logarithmic sine sweep

$$V_{ss,e} = \left\{ \sum_{i=1}^{m} \left[\frac{\ln\left(\frac{f_{2e}}{f_{1e}}\right)}{\ln\left(\frac{f_{2i}}{f_{1i}}\right)} \right] \left(\frac{t_i}{t_e}\right) \left(V_{ss,i}^b\right) \right\}^{1/b} \tag{3-70}$$

where f_{2i} and f_{1i} set upper and lower frequency boundaries on the original sweep, and f_{2e} and f_{1e} reflect chosen boundaries for the equivalence sweep vibration, assuming that both the original sweep and the equivalent sweep pass through specimen resonances of interest.

Several random levels to one random level

$$W[V]_e = \left\{ \sum_{i=1}^{m} \left(\frac{t_i}{t_e}\right) W[V]_i^{b/2} \right\}^{2/b} \tag{3-71}$$

The expressions for transformations between types of vibration history while holding exposure time constant used a factor ϕ to simplify computation and reduce computer time.

$$\phi = \frac{\ln \frac{f_2}{f_1} NDQ}{f_n t} \approx \left(\frac{1.89}{b}\right)^{0.645} \tag{3-72}$$

where ϕ was shown to introduce no more than 2% error when $10 \geqslant Q \geqslant 30$ and $10 \geqslant f_n \geqslant 1500$ Hz.

From a logarithmic sine sweep to a sine dwell

$$V_{sd,e}^{ss} = V_{ss,e} \left[\frac{\phi}{Q \ln\left(\frac{f_{2e}}{f_{1e}}\right)} \right]^{1/b} \tag{3-73}$$

From a sine dwell to a logarithmic sine sweep

$$V_{ss,e}^{sd} = V_{sd,e} \left[\frac{Q \ln\left(\frac{f_{2e}}{f_{1e}}\right)}{\phi} \right]^{1/b} \tag{3-74}$$

From a random spectrum to a logarithmic sine sweep

$$V_{ss,e}^{r} = \left(\frac{W[V]_e \pi f_n}{Q}\right)^{1/2} \left[\frac{Q \ln\left(\frac{f_{2e}}{f_{1e}}\right) \Gamma\left(1 + \frac{b}{2}\right)}{\phi}\right]^{1/b} \tag{3-75}$$

From a random spectrum to a sine dwell

$$V_{sd,e}^{r} = \left(\frac{W[V]_e \pi f_n}{Q}\right)^{1/2} \left[\Gamma\left(1 + \frac{b}{2}\right)\right]^{1/b} \tag{3-76}$$

From a logarithmic sine sweep to a random spectrum

$$W[V]_e^{ss} = \left(\frac{V_{ss,e}^2 Q}{\pi f_n}\right) \left[\frac{\phi}{Q \ln\left(\frac{f_{2e}}{f_{1e}}\right) \Gamma\left(1 + \frac{b}{2}\right)}\right]^{2/b} \tag{3-77}$$

From a sine dwell to a random spectrum

$$W[V]_e^{sd} = \left(\frac{Q V_{sd,e}^2}{\pi f_n}\right) \left[\frac{1}{\Gamma\left(1 + \frac{b}{2}\right)}\right]^{2/b} \tag{3-78}$$

The final equivalent level is derived by combining all similar types of vibration.

Final random equivalent

$$W[V]_f = \left\{\left(W[V]_e^{r}\right)^{b/2} + \left(W[V]_e^{ss}\right)^{b/2} + \left(W[V]_e^{sd}\right)^{b/2}\right\}^{2/b} \tag{3-79}$$

Final logarithmic sine sweep equivalent

$$V_{ss,f} = \left[\left(V_{ss,e}\right)^b + \left(V_{ss,e}^{r}\right)^b + \left(V_{ss,e}^{sd}\right)^b\right]^{1/b} \tag{3-80}$$

Final sine dwell equivalent

$$V_{sd,f} = \left[\left(V_{sd,e}\right)^b + \left(V_{sd,e}^{ss}\right)^b + \left(V_{sd,e}^{r}\right)^b\right]^{1/b} \tag{3-81}$$

The approach presented by Gerks may be adapted for use on a computer. When so doing, the computer output may be displayed as an equivalent test spectrum plotted directly as a family of curves which are a function of Q. As an example, Fig. 3-15 illustrates the result of combining both random and sine-sweep spectra into an equivalent sine-sweep test. If the specimen is resonant at the four frequencies indicated in Fig. 3-15, then a final test envelope is represented by the dashed line. The construction of the dashed line was made possible by the prior assumption that significant damage occurs only in a resonant region.

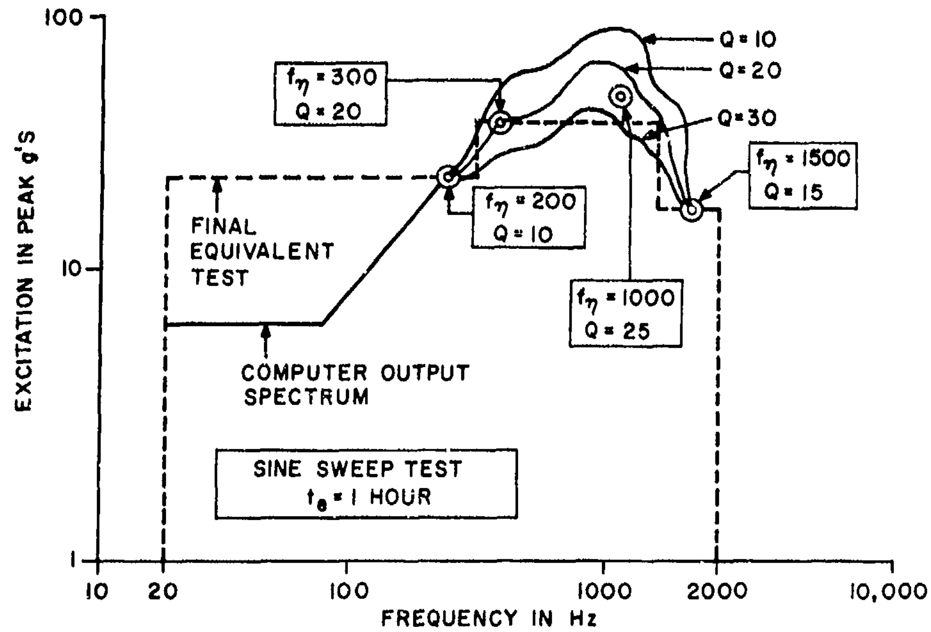


Fig. 3-15. Example determination of an equivalent vibration envelope.

Change-of-Test-Type Practices

To perform change-of-test-type equivalence we have seen that it is necessary to

1. Determine or estimate the response frequencies and associated resonance amplification factors which describe the test specimen.

2. Describe the original test or field history vibration in terms of a temporal sequence of sinusoidal, random, sine-sweep, or a combination thereof, vibration loading.

3. Select an appropriate S-N or $\bar{S}$-N curve or curves which account for variable factors such as stress ratio, mean stress, residual stresses, temperature, notches, and the like.

4. Make comparisons between vibration h...ories or perform equivalence on the basis of exciting the specimen by a mounting structure of known impedance.

5. Use a suitably slow sweep rate when determining a sinusoidal sweep equivalent test, so that specimen responses to an excitation can build up to equal levels as if excited by the original vibration history.

6. Review the equivalent test relevant to structural and operational limitations, such as level sensitivity and range of linear response, to assure the validity of the equivalence.

7. Recognize that an equivalent test is a result of both original vibration history and specimen dynamic structural characteristics.

Thus a test which is equivalent for one type of design is not necessarily or usually equivalent for another type of design, even though identical original vibration-excitation histories were assumed.

The change-of-test-type equivalences are summarized in Table 3-4. They have many similar characteristics. The simpler approaches require only very general and less realistic assumptions to retain validity. The more complex and complete approaches, such as presented by Gerks, lead to improved accuracy.

Table 3-4. A Summary of Change-of-Test-Type Equivalence Practices

Quantity	*Author*	*Description*
S_e	Miles	Equivalent stresses were found for a linear SDF oscillator based on equal damage using Miner's theory for two types of loading: 1. Monotonically decreasing loads (Eq. (3-31)) 2. Peaked (Gaussian) load spectrum (Eq. (3-32))
S_e, $V^r_{sd,e}$, $\bar{V}^{ss}_{sd,e}$, $t^r_{sd,e}$	Spence and Luhrs	Extends Miles' analysis to find equivalent sine-dwell or sine-sweep excitation levels for two types of original loading spectra: 1. Random only. 2. Random plus sine sweep. The computational sequence is summarized by Table 3.3.
S_e	Crede and Lunney	Use Miles' analysis to find an equivalent stress for the case of beat excitation. See Eqs. (3-48) and (3-49).

Table 3-4 (Continued)

Quantity	*Author*	*Description*
All major parameters	Mains	A generalized approach to damage equivalence using Miner's theory yields a set of relationships for all major parameters involved in vibration scaling. A summary of the expressions is given in Table 3-4.
Relates sine-dwell, sine-sweep, and random excitation spectra	Hall and Waterman	A set of three equivalence pairs created for two cases: 1. Equal amounts of work expended due to damping forces 1.1. Sine dwell–random (Eq. (3-58)) 1.2. Sine sweep–random (Eq. (3-59)) 1.3. Sine sweep–sine dwell (Eq. (3-61)) 2. Similar to above except uses a plot of actual specimen amplitude vs time to failure to find damage functions 2.1. Sine dwell–random (Eq. (3-66)) 2.2. Sine sweep–random (Eq. (3-67)) 2.3. Sine sweep–sine dwell (Eq. (3-68))
Relates sine-sweep, sine-dwell, and random excitation spectra	Gerks	A complete set of relationships resulting from a Miles type of analysis and Miner's theory. The reduction procedure was illustrated by Fig. 3-14, and the necessary expressions were given in Eqs. (3-69) through (3-81).

CHAPTER 4
MAGNITUDE EQUIVALENCES

4.1 Preliminary Considerations

The magnitude equivalences constitute the second major category of the damage-based vibration equivalences. Two types of magnitude equivalence are discussed: equivalence of stress amplitudes, and equivalence by virtue of like malfunction. There is yet another category, that of the probability that specimen elements will collide. Such a computation may be treated as a first-passage problem [8] and is not covered in this monograph.

A common shortcoming of both the malfunction and cumulative-damage equivalences is that they were often without concern for the influence of mounting interface impedance. They are not totally independent of driving-force coupling because some of these equivalence techniques are predicated on the use of specimen response data. The interface impedance problem is discussed in Chapter 5.

Several of the magnitude equivalences were developed to establish a common reference between the effects caused by sinusoidal and random vibration spectra. They appeared at a time when random vibration concepts were first being introduced. At that time many engineers wanted a simple sinusoid-random equivalence correlation so they could apply sinusoidal testing experience to the "new" random vibration environment.

Work was also undertaken to find an equivalence between narrowband swept random and wideband random testing. The need for such an equivalence had a practical basis in that the swept random spectrum would demand less drive power than the wideband excitation. The reduced power requirement for the narrowband swept random test was achieved by not simultaneously exciting the entire wideband spectrum; however, the actual test time was increased as a result of this test approach.

The magnitude equivalences were formed with the intent of using parameters which were less illusive to the intuitive designer than the parameters required for the cumulative-damage equivalences. Under limited circumstances the magnitude equivalences have proven valuable from the standpoint of providing insight into specimen responses to various types of excitation. Unfortunately, the attempts to achieve correlation based on specimen performance degradation, or malfunction, did not yield valid equivalence techniques.

The majority of the magnitude equivalences are limited to a unity time scaling between various types of vibratory excitation. Some of the equal-motion equivalences provide a method for changing the time scale, so that they could be used for the purpose of deriving an accelerated test.

4.2 Stress Equivalences

The equal-stress equivalences were developed mostly from mathematical relationships used to describe Gaussian processes and the classical SDF linear oscillator. An equality was assumed to exist between various vibration experiences when each of these experiences would produce like sets of stress amplitudes and a like number of occurrences at each stress amplitude. It was further assumed that a defined relationship existed between a specimen's response amplitude and the stress resulting from the response amplitude.

One basis for correlation involved equating the rms response between different types of vibration excitation. Another basis of correlation was to equate the distribution of response acceleration peaks above some level or at a known level which would cause failure or malfunction. The equal rms response and equal distribution of peaks theories were used to form the following substitutions:

1. Sinusoidal dwells of a single frequency or several discrete frequencies for random excitation.
2. Sinusoidal sweep testing for random excitation.
3. Swept narrowband random for wideband random excitation.

Recognition of the importance of factors such as resonant amplification Q and natural frequency f_n was provided by the stress-amplitude equivalence techniques. In certain cases, the stress equivalence eventually related to the concept of specimen damage. As shown by Patrick [71], this eventual relationship to damage was expected because the concept of damage is explicit in time, and both the time-to-failure predictions and time-scaling techniques are important facets of equivalence technology.

As in the application of the other equivalence techniques, it is advisable to apply the stress-equivalence relationships with caution. If the fundamental assumptions and shortcomings are understood for each technique, these equivalences may be used as additional design or diagnostic aids.

The stress equivalences are divided into the categories of equal rms response, equal distribution of peaks, and swept random.

Equal RMS Response

Certain measurements are made and used to characterize a random vibration excitation or response spectrum. Either instantaneous or peak values of acceleration may be observed or recorded; however, the vibration spectrum is conventionally defined by a plot of power spectral density, $W[V]$, vs frequency. A true rms meter is frequently used to determine the rms acceleration value $\bar{V}_r$. When the rms acceleration level is assumed to be constant over a narrow frequency band Δf, the power spectral density (PSD) in g^2/Hz is simply

$$W[V] = \left(\frac{(\bar{V}_r)^2}{\Delta f}\right). \qquad (4\text{-}1)$$

where Δf is the bandwidth of the filter or meter used to measure $\bar{V}_r$, and V is frequency dependent.

When vibration test data are narrowband, a condition which may result from the filtering caused by a local structural resonance, it is convenient to deal with peak measurements. The acceleration peaks are considered to be Rayleigh distributed in the majority of the following equivalences; however, they are in theory Rayleigh distributed only in the limit as the response bandwidth tends toward zero. This simplifying assumption is usually valid because the filter bandwidth used in performing a spectral analysis is usually very small compared with the center frequency of the analyzed bandwidth. Broadband signal acceleration peaks are assumed to be Gaussian distributed.

A comparison of the normal or Gaussian distribution in the Rayleigh distribution is shown in Fig. 4-1. Although in many instances the assumptions of a normal distribution and applicability of the Rayleigh distribution are good representations of actual data, this is not always true. As noted by Mustain [72] and others, actual field data will sometimes yield varying bandwidths. Thus the peak distributions will deviate from the Rayleigh distribution, particularly as structural nonlinearities become effective.

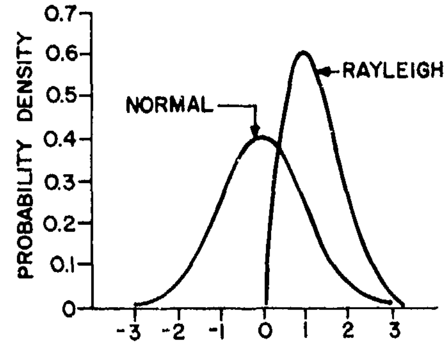


Fig. 4-1. A comparison of normal and Rayleigh distributions.

For the general case where a random input acceleration level is known to vary with frequency,

$$[\bar{V}_r(\text{response})]^2 = \frac{1}{2\pi}\int_0^\infty W[V(\omega)]\,|H(\omega)|^2 d\omega, \qquad (4\text{-}2)$$

where $H(\omega)$ defines the ratio of response acceleration to input acceleration as a function of frequency.

The rms response of an SDF linear oscillator to a white random input signal was given by Spence [73] as

$$\bar{V}_r(\text{response}) = \left(\frac{\pi}{2} W[V] f_n Q\right)^{0.5}, \qquad (4\text{-}3)$$

where the resonant amplification or quality factor Q was classically [74] represented as

$$Q = \frac{1}{2\delta} = \frac{f_n}{B} , \tag{4-4}$$

with δ representing the ratio of actual damping to critical damping and B the half-power bandwidth.

If a linear system has several distinct and well separated resonances, the overall response to a white random input is

$$V_r(\text{response}) = \left(\sum_{i=1}^{m} \frac{W[V_i]\pi f_{ni} Q_i}{2}\right)^{0.5} , \tag{4-5}$$

where the assumption of white random input need be valid only in the vicinity of each natural resonance.

The response of a linear oscillator to a sinusoidal input excitation at resonance is

$$V_{sd}(\text{response}) = V_{sd}Q , \tag{4-6}$$

where V_{sd} is the rms input acceleration.

The equivalent sinusoidal rms input acceleration which will represent the random excitation on an equal rms response basis is found by equating the rms responses described by Eq. (4-3) and (4-6):

$$V_{sd} = \left(\frac{\pi}{2} \frac{f_n W[V]}{Q}\right)^{0.5} = V_{sd}^r , \tag{4-7}$$

where V_{sd}^r is the sinusoidal rms input excitation equivalent of the random input excitation, and f_n/Q is the bandwidth B.

The peak sinusoidal acceleration level is usually [71,75-78] taken as

$$V_{sd,e}^r = 3V_{sd}^r , \tag{4-8}$$

where $V_{sd,e}^r$ is the peak sinusoidal input equivalent of the random input acceleration. The factor 3 used in Eq. (4-8) comes from stochastic theory and is a conventionally used boundary between frequent and infrequent occurrences in a Gaussian distribution. V_{sd}^r may be mathematically related to one standard deviation σ by

$$\sigma = V_{sd}^r , \tag{4-9}$$

which was plotted vs probability of occurrence in Fig. 4-2.

When a specimen has several response frequencies, each is considered on an individual basis. An equivalent test would involve a sinusoidal dwell at each response frequency of interest.

An equal rms response equivalence described by Trotter [75] provides a sinusoidal input at each resonance frequency such that the rms response at each resonance is equated to the overall rms response,

$$[\bar{V}^r_{sd,e}]_k = \frac{1}{Q_k}\left(\sum_{i=1}^{m} \frac{\pi}{2} f_{ni} W[V_i] Q_i\right)^{0.5} , \tag{4-10}$$

where the subscript k refers to the conditions associated with the kth sinusoidal response. This approach tends to yield lower values of a computed sinusoidal equivalent excitation than the values computed by use of Eq. (4-7). The lower values result from the use of the overall $W[V]$ rather than the $W[V]$ at each specific resonance. This approach would tend to average the sinusoidal equivalent excitation levels. Thus when a specimen has one resonance which is dominant over other resonances, the lower resonances would be somewhat overtested, and the dominant resonance would be significantly undertested. For these reasons the results obtained by the application of Eq. (4-10) would yield a less satisfactory basis for equivalence than the results obtained by the use of Eq. (4-7). In general, a single-frequency sinusoidal dwell equivalence of a random spectrum can be derived only for the immediate region of a resonance. The type of failure may be different at each resonance; i.e., wear, fatigue, or performance malfunction.

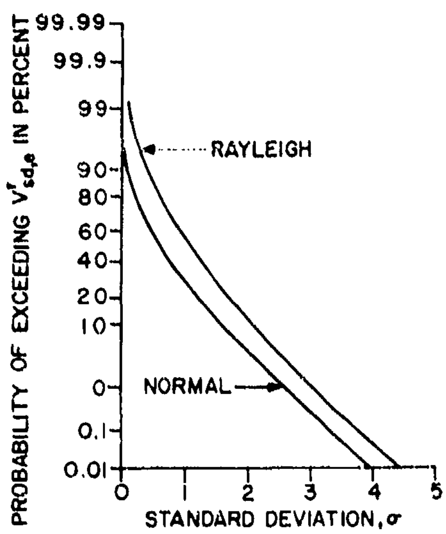


Fig. 4-2. Probability of exceeding certain levels in normal and Rayleigh distributions.

The equal rms response techniques may be used to design an equivalent test which would consist of a sinusoidal sweep substitute for the random excitation. Caution is advised, however, because such attempts may be fraught with problems [77]. That is, an equivalent sinusoidal sweep would involve the adjustment of both excitation amplitude and sweep rate as a function of excitation frequency. Morrow and Muchmore [78] concluded that there is no mathematically valid way of providing a conventional single-frequency sweep that is equivalent to a continuous spectrum for all types of failures.

Equal Distribution of Peaks

A technique for scaling test time between a long-duration, relatively low-intensity and a short-duration, relatively high-intensity random excitation was introduced by Curtis [79]. He equated acceleration response peaks above some level. Blake and Oleson [80] and Oleson [81] also suggested that test equivalence exists when there is a similar distribution of response stress peaks at high levels.

The assumptions necessary for the formation of an equal distribution of peaks equivalence include the following:

1. The resonant transmissibility Q is known or can be assumed.
2. Damage occurs primarily due to the excitation of resonance. Thus the analysis is focused on narrow frequency bands, and the results of the analysis are influenced by the expected resonant characteristics of the specimen.
3. The narrowband vibration probability density function for the distribution of peaks is a Rayleigh distribution.
4. The specimen's fragility is known in terms of a maximum input acceleration level which will cause no damage vs frequency.
5. Damage is produced only by acceleration peaks above the fragility-level input acceleration V_f.

The distribution of peaks for two narrowband vibration histories of different rms acceleration level and some center frequency is shown in Fig. 4-3. Equality is assumed when the rms acceleration of each vibration history is maintained for a time ratio such that the fragility-level acceleration is exceeded an equal number of times. That is,

$$t_1 = K t_2 \ , \tag{4-11}$$

where t_1 and t_2 represent the time duration of vibration history 1 and vibration history 2, and K represents the ratio of the shaded areas of the curves in Fig. 4-3.

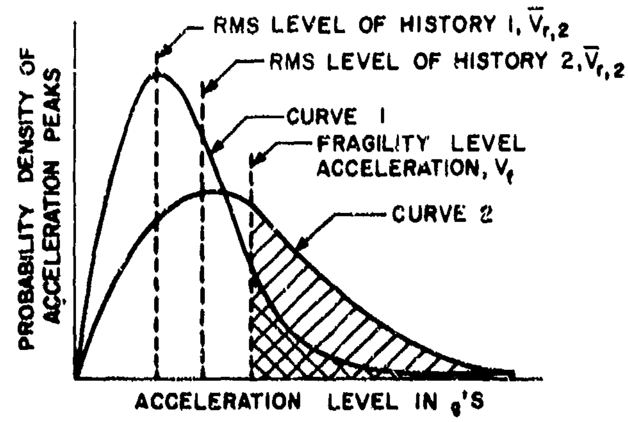


Fig. 4-3. A comparison of two narrowband vibration histories [79].

Application of assumption number 3 to compute the number of peaks exceeding some level V_f yields

$$K = \frac{t_1}{t_2} = \exp\left\{\frac{V_f^2}{2\bar{V}_{r,1}^2}\left(1 - \frac{W[V_1]}{W[V_2]}\right)\right\}, \tag{4-12}$$

where $\bar{V}_{r,1}$, the in-band rms acceleration, is $(f_n W[V_i]/Q)^{0.5}$, and $W[V_1]$ and $W[V_2]$ are evaluated at the natural response frequency of interest. Equation (4-12) was plotted by Curtis [79] and appears as Fig. 4-4.

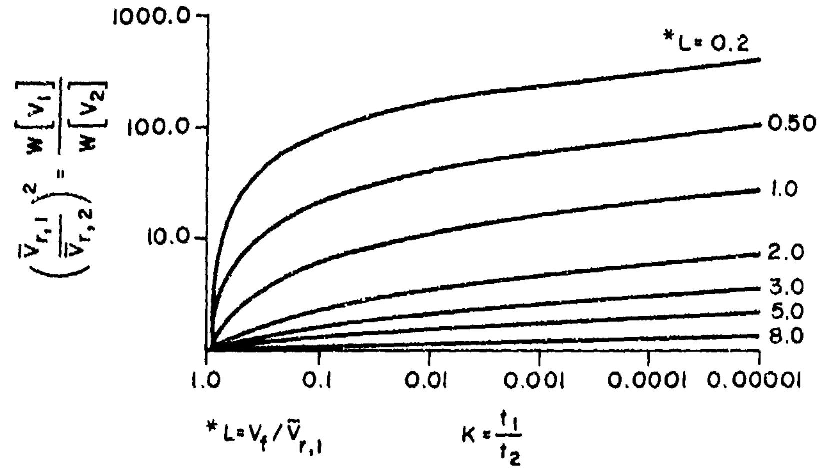


Fig. 4-4. Time vs the ratio of mean square acceleration for several values of $\bar{V}_f/V_{r,1}$ [79].

Application of the equal distribution of peaks method is accomplished in four steps.

First, the fragility envelope must be obtained from available test data, or by assumption based on analyses of similar specimens and perhaps the use of analytical modeling techniques. Second, the in-band rms acceleration response is computed,

$$\bar{V}_{r,1}\,(\text{response}) = \left(\frac{\pi}{2} W[V_1] f_n Q_1\right)^{0.5} . \tag{4-13}$$

Third, the value of k is determined from Fig. 4-4. Fourth, the test duration, assuming a comparison of like narrowband envelopes, is

$$t_{e,1} = t_1 + K t_2 , \tag{4-14}$$

where $t_{e,1}$ is the equivalent test duration at level 1 for the combined vibration histories at levels 1 and 2.

When several narrowband envelopes exist, such as expected in a complex specimen excited by a wideband random signal, Eq. (4-14) is used at each frequency of interest. As a result, the individual total test times will vary with frequency, which is a weakness of the method because some average value of time must eventually be selected to arrive at the narrowband excitation approximation of the broadband excitation.

Swept Random

Booth [82] and Broch [83] extended the equal distribution of peaks equivalence techniques to the problem of substituting an intense swept narrowband random signal for a wideband random signal. The major advantage of a swept narrowband random equivalent test is that a vibration exciter of smaller force capability will suffice, whereas a much larger exciter would be required for the wideband test. Also, swept random testing complements wideband testing in that resonances are excited sequentially. As a result of sequential excitation, any frequency band in which specimen failure or malfunction appears is readily isolated, a fact which expedites subsequent failure analysis efforts.

The following characteristics of the wideband excitation must be duplicated by the swept narrowband signal:

1. The total number of acceleration peaks must remain the same inside each resonance band.
2. The rms test level must be adjusted to provide the same number of acceleration peaks in any interval of acceleration level.
3. The probability distribution of peaks with respect to the rms test level must match for both types of test.

The first characteristic is accommodated by use of a logarithmic sweep rate. The changing sweep rate is necessary because the bandwidth of a resonance, when using any single value of Q, increases with increasing frequency (recall Eq. (4-4)). The second characteristic is accommodated if the rms magnitude of the narrowband excitation is increased as the square root of frequency, i.e., 3 dB/octave (recall Eq. (4-3)).

To facilitate the implementation of a test which has tolerances on the input acceleration level, Booth [82] introduced a factor called the acceleration gradient. The acceleration gradient is simply the instantaneous acceleration divided by the square root of radial frequency,

$$AG = \frac{\bar{V}_r}{\sqrt{2\pi f}}, \tag{4-15}$$

where AG is the acceleration gradient and $\bar{V}_r$ is a function of the bandwidth of the meter used to monitor the input acceleration. Now, rather than specifying a test in terms of a PSD-vs-frequency spectrum, the test would be specified in terms of a constant acceleration gradient.

A swept random test is shown in Fig. 4-5, and a simplified sweep random test system is shown in Fig. 4-6. In Fig. 4-6 it is assumed that the meter will read *AG* directly, and the *AG* signal is used to control the sweep random generator. The random generator in turn controls the power amplifier.

The third characteristic, that of matching the probability distribution of peaks between the swept random and wideband spectrum, is more difficult. Booth simplified the problem by constructing a plot of the probability that response peaks in a Rayleigh distribution will exceed some level V_1 where $\bar{\lambda}$ is the ratio

$$\bar{\lambda} = \frac{V_i}{\bar{V}_r}, \tag{4-16}$$

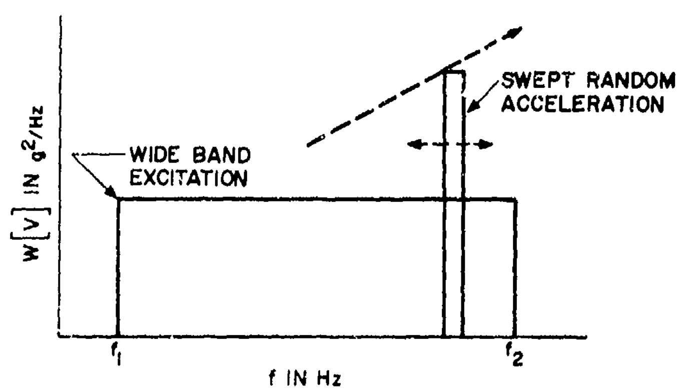


Fig. 4-5. A comparison of wideband and swept narrowband random vibration excitation.

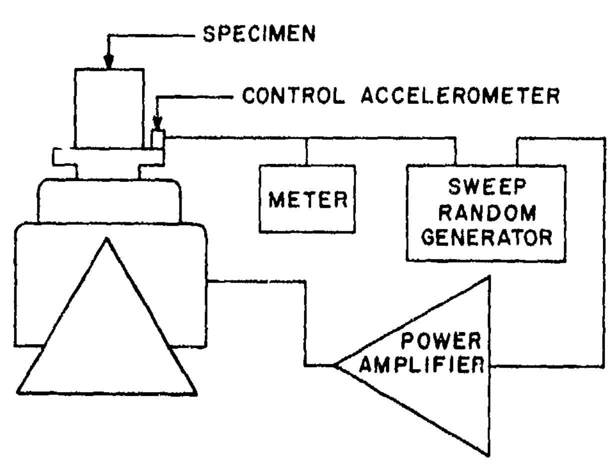


Fig. 4-6. A basic sweep random test diagram.

and V_i is the magnitude of the ith-level peak acceleration. The probability plot for $\bar{\lambda}$ is given as Fig. 4-7. Next, it is necessary to determine the accumulated peak probability distribution for the swept random signal based on the resonance characteristics of the system. This may be accomplished by a numerical integration procedure [82] or by the use of analog models [83]. The new curve is plotted to the same scale as Fig. 4-7 using

$$\bar{\lambda}^* = \frac{V_i^*}{Q\bar{V}_r^*}, \tag{4-17}$$

where the asterisk denotes the actual response conditions. The two plots are overlaid as shown in Fig. 4-8 to determine suitable vertical- and horizontal-axis scale factors. The scale factors can be used to determine the desired excitation level and sweep time for a practical sweep random test by the procedure outlined in Table 4-1. The horizontal scale factor q is taken as

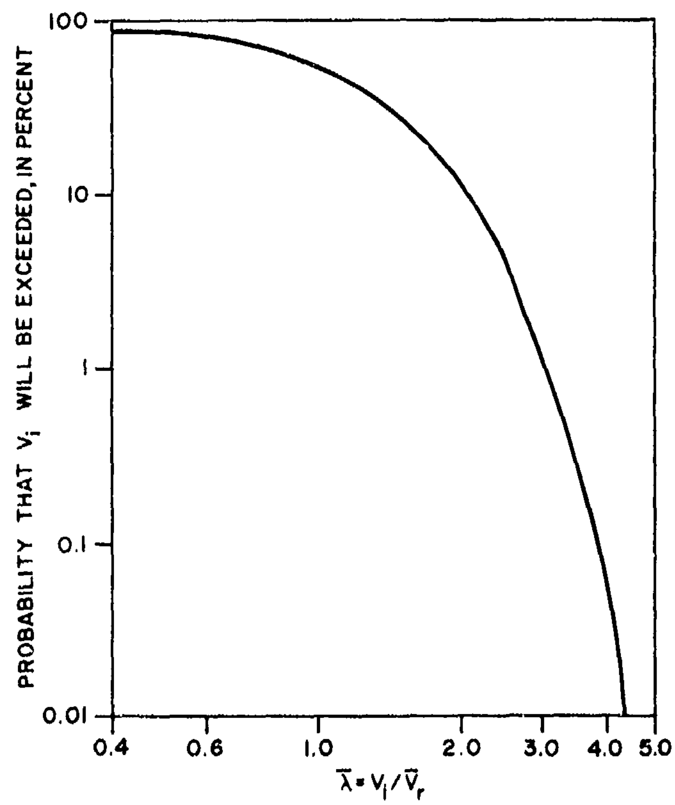


Fig. 4-7. Probability that an acceleration peak will have a magnitude greater than V_i.

$$q = \frac{\bar{\lambda}^*}{\bar{\lambda}}, \tag{4-18}$$

and the vertical scale factor s as

$$s = \frac{P[\bar{\lambda}^*]}{P[\bar{\lambda}]}. \tag{4-19}$$

A possible drawback of the swept random equivalent testing technique is that a relatively long test time is required to achieve the same damage-producing capability as the original wideband test [84].

The test time may be shortened by the following techniques, also suggested by Booth:

1. Use several signal generators, each of which would simultaneously sweep portions of the total frequency spectrum.

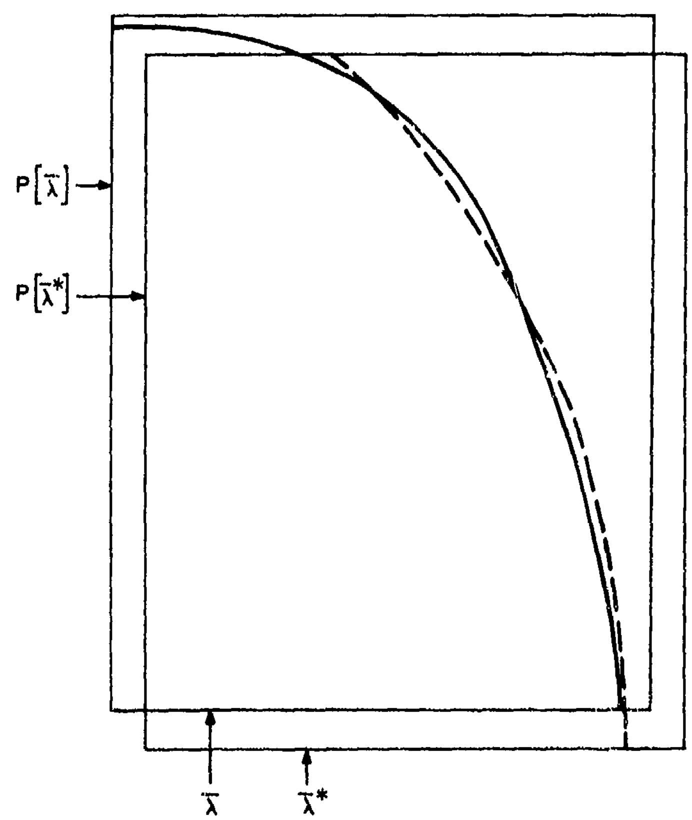


Fig. 4-8. An overlay of theoretical (test-generated) and predicted (actual) peak response probability curves.

Table 4-1. Procedure for Computing a Narrowband Swept-Random Equivalent of a Wideband Random Excitation

Step	*Approach*
1. Define the initial wideband input spectrum	1. Parameters required are frequency range, total test time, and $\bar{V}_r$, where $$\bar{V}_r = \{W[V](f_2 - f_1)\}^{0.5} \qquad (4\text{-}1)$$
2. Determine the equivalent rms sweep acceleration level	2. Plot peak probability distributions and overlay as in Fig. 4-8. Find the scale factor q of the horizontal axis positions. Then $$\bar{V}_{r,e} = q\left(\frac{\pi}{2Q} W[V] f_n\right)^{0.5} \qquad (4\text{-}7)$$ defines the equivalent rms acceleration level. Use f_2 from Fig. 4-5 to compute the maximum value of $\bar{V}_{r,e}$.
3. Compute the acceleration gradient AG for test control purposes	3. Combine Eqs. (4-7) and (4-15): $$AG = \frac{\bar{V}_r}{\sqrt{2\pi f}} = q\sqrt{\frac{W[V]}{4Q}}$$
4. Compute test duration T	4. Determine vertical-axis scale factor s from overlay plot of Step 2. Then $$t = sQt_\omega \ln\left(\frac{f_2}{f_1}\right), \qquad (4\text{-}20)$$ where t_W = original wideband test time.
5. Define final swept narrowband random test parameters	5. Use the following steps: 5.1 Compute $\bar{V}_{r,e}$ by Step 2 at the highest frequency f_2, and apply at 3 dB/octave rolloff as shown in Fig. 4-5 5.2 The constant AG for the test as determined by Step 3 5.3 The total test time as found by Step 4

2. Provide automatic level regulation to compress some of the higher peaks in the narrowband excitation signal. The result is an increase in the number of peaks per unit time without an increase in test level. A new acceleration peak distribution plot is necessary to describe each change in peak compression. By using the new set of peak distribution plots to form a family of overlays, as in Fig. 4-8, new scale factors q and s are found and used to change the test time as computed per Table 4-1.

Broch [83] expanded the change of test time procedure by defining a term β:

$$\beta = \frac{\text{Compressor speed in dB/sec}}{\text{Bandwidth in Hz}} , \tag{4-21}$$

where the faster compressor speeds mean more peak compression. The bandwidth refers to the bandwidth of the sweeping narrowband signal. The effect of β on a peak-acceleration distribution curve is shown in Fig. 4-9. Broch also plotted a summary of many matches of peak-acceleration probability-density curves to the Rayleigh curve. Broch's plot is given as Fig. 4-10. He does not recommend values of β in excess of 300 because the higher compressor speeds will cause waveform distortions at the lower frequencies and yield unsatisfactory acceleration-amplitude probability distributions.

4.3 Malfunction-Based Equivalences

The assumed basis for the formulation of the malfunction equivalences was that each type of vibration would produce correlative amounts of specimen performance degradation. In each case a general equivalence was sought without consideration of the construction details of the specimen. The concept involved was that complex devices, unlike simple mechanical structures, suffer performance changes at vibration test levels which are lower than vibration levels that lead to structural or catastrophic failure. The term *vibration fragility* was expanded to include the threshold excitation conditions which produce specimen malfunction. Specimen malfunction was considered to be a reversible process wherein specimen operation exceeds performance tolerances during load application, and the specimen would return to a condition of satisfactory operation after the loading was removed. It follows from the definition of malfunction that equipment fragility was not considered to be time dependent, but was somehow related only to acceleration level.

The malfunction-based equivalences were investigated in an early and unsuccessful attempt to equate the sinusoidal and random vibration environments. A brief review of these attempts to form a malfunction equivalence will illustrate the logical fallacies involved and as such will be useful in identifying and avoiding these fallacies in future work.

Foster [85] investigated the feasibility of forming a malfunction equivalence between the random and sinusoidal excitation of complex electronic equipment.

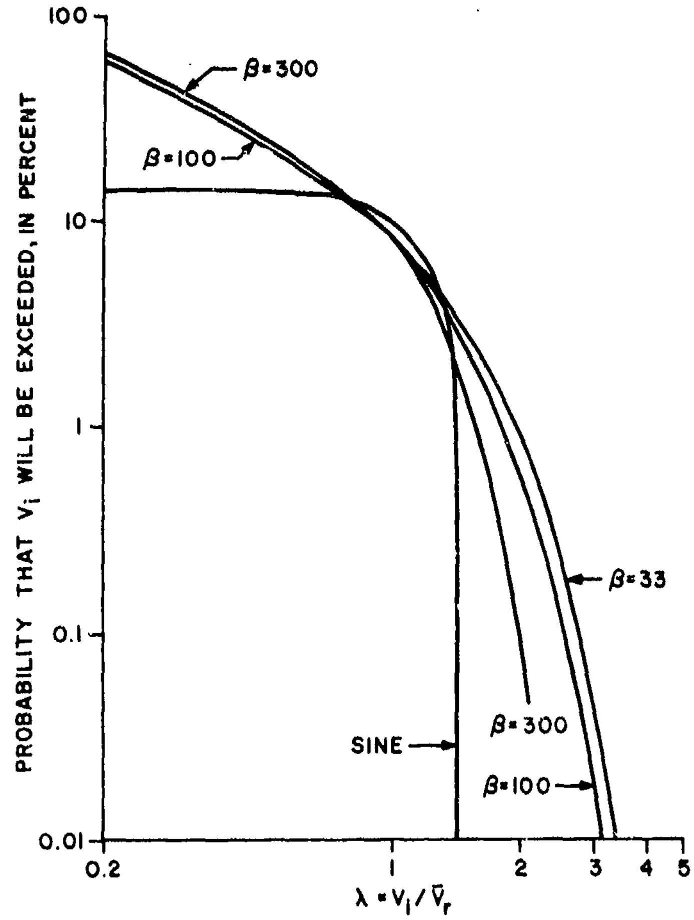


Fig. 4-9. Probability that an acceleration peak will have a magnitude greater than V_i, using data from [80].

He selected a military-quality state-of-the-art (in 1961) transmitter-receiver and a commercial aircraft horizon-indicator instrument as test specimens. His testing program involved the excitation of the specimens by a random spectrum of constant spectral density in the 20- to 2000-Hz bandwidth, and a very slow sinusoidal sweep at constant peak g level in the same frequency interval.

The specimens were tested at random PSD levels, stepwise, from 0.001 to 0.010 g^2/Hz; and at sinusoidal levels from ± 0.25 to ± 1.50 g. The specimens were hard mounted to the electrodynamic exciter and tested in three mutually perpendicular axes. Electrical degradation of specimen output signals was the criterion for correlation.

Foster concluded that no constant quantity could be found to relate electrical degradation between sinusoidal and random excitation. However, he did make two observations:

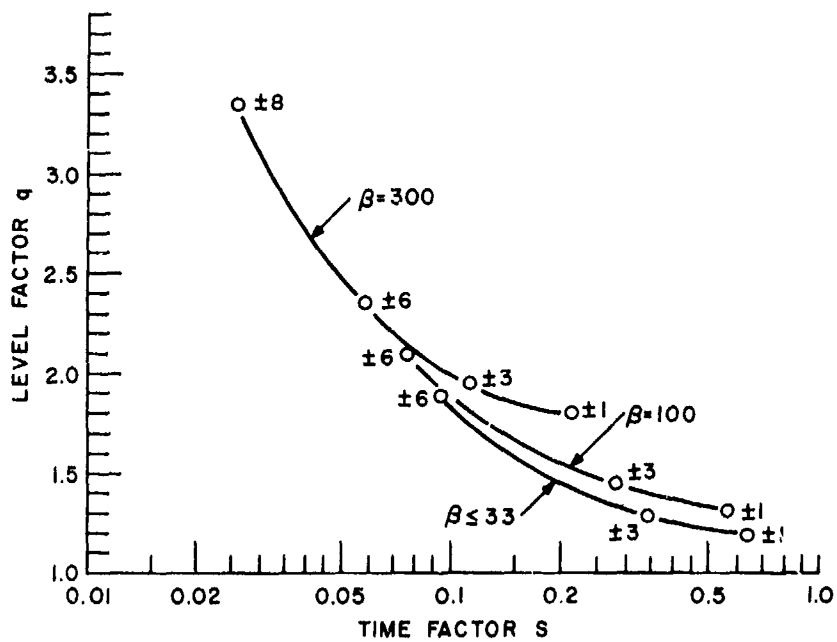


Fig. 4-10. Time-level exchange curve for the selection of level factors q and time factors s [83]. The accuracy of the match between the new peak distribution and the Rayleigh curve in the 2σ to 3σ region is noted in decibels by each plotted point.

1. Low levels of sinusoidal excitation produced higher levels of electrical noise output than did relatively high levels of random input. The electrical signal distortion was higher as a result of ± 0.25-g sinusoidal testing than for a random test level of 0.010 g^2/Hz over the 20- to 2000-Hz band.

2. The slope of a set of curves plotted to display specimen performance signal degradation vs test level tended to vary with excitation frequency and axis of excitation. Also, when the signal degradation resulted from several simultaneous response modes, correlation was not possible because no single sinusoidal excitation frequency gave a direct indication of signal degradation under random excitation.

Brust [86] performed an investigation similar to that of Foster. However, his specimens included light bulbs, an airspeed indicator, a mechanical chopper, and a motor-driven resolver. He concluded that sine-random equivalence does not exist on a malfunction basis. His results show that sinusoidal testing can yield false indications of the severity of the random environment when the multimode responses, such as caused by random excitation, are considered.

Curtis and Abstein [87] also conducted an investigation to establish a correlation between the effects of random and sinusoidal excitation. They used as specimens the electronic packages from a GAR-4 Falcon missile. The functional performance of three nominally identical units was found to vary considerably under random excitation. In one case catastrophic failures occurred prior to functional failures. There were distinct differences in the nature of specimen malfunction between each type of vibration excitation. Curtis and Abstein concluded:

1. In certain cases, no correlation exists.
2. If a correlation exists for complex systems, it will be so complicated that both types of testing are required anyhow, which negates the need for the correlation.
3. It is possible that correlation may exist for a very simple type of malfunction such as wiper-arm chatter in a servopositioner.

The work on malfunction equivalences was conducted in the early 1960's and due to a lack of encouraging results, has not been seriously considered in recent years.

4.4 Use of Magnitude Equivalences

The magnitude equivalences are a valuable addition to the cumulative-fatigue and mechanical-impedance equivalences. A common characteristic of the magnitude equivalences is that they ignore the mounting impedance problem. This characteristic is unfortunate in an absolute sense, but it is not a limitation once a specific test has been described. It causes no restriction when comparing two different tests on one specimen, both of which involve mounting fixtures of great stiffness or of equal mechanical impedance.

Some caution is advised in the application of the magnitude equivalences. First, using the PSD-vs-frequency plot only to describe a random environment does not give any information about the distribution of acceleration peaks. Thus a non-Gaussian process would be poorly represented on an equal rms response basis if the envelope of peak amplitudes were not considered. Second, it was noted by Warren [88] that, although the response of a system to a sinusoidal signal is proportional to Q (Eq. (4-6)) and the response to a random signal is proportional to $Q^{0.5}$ (Eq. (4-5)), actual flight records show that response varies as Q^n, where $0.7 < n < 0.8$. Third, no history of vibration effects was included, i.e., cumulative damage, which would be of significance if long test periods were anticipated. Fourth, unless there is a very detailed knowledge of the specimen's response characteristics, it is probably inadvisable to attempt to use a sinusoidal single-frequency signal to duplicate an overall performance degradation due to a wideband excitation. We need to know how specimen performance relates to each response and how these individual performance degradations combine. Fifth, the magnitude equivalences tend to ignore specimen damping characteristics. Thus when a specimen is heavily damped significant errors are possible.

Foster [85] plotted signal degradation vs input acceleration level for various types of input spectra. Although his results provided a nonlinear relationship between performance and input excitation level, curves of this nature could be obtained experimentally and used for performance comparisons between different types of tests.

CHAPTER 5
INTERACTION EQUIVALENCES

5.1 Preliminary Considerations

The interaction equivalences represent a most important area of vibration equivalence, an area which must be considered regardless of the type of damage-based equivalence technique which may be selected and applied to any specific situation. The interaction equivalences may be described more accurately as techniques which are used to improve test realism rather than true equivalences.

The interaction equivalences were divided into two areas for discussion purposes, although neither area is separable from the other. The two areas include the consideration of complex-motion and mechanical-impedance concepts.

Complex-motion equivalence means that proper allowance and recognition is given the entire situation within which a specimen has been operated or will be expected to operate under field and laboratory conditions. Complex-motion equivalences become of increasing importance as the size and complexity of a specimen, its mounting situation, excitation force spectra, and force location patterns become more complex.

As an example of moderately complex specimens, consider a shelf of electronic equipment which may be mounted in any one of several aircraft. Each carrier aircraft is powered by different engines located at somewhat different locations on an airframe of a different structural design. In addition, each aircraft would be expected to vary in airspeed, maneuverability, and general usage, resulting in a variance in the aerodynamically induced force systems which act on each airframe. The shelf of electronic equipment, which is attached, for example, to each airframe by four common attachment points, will receive a very different vibration experience in each aircraft. This vibration experience will differ in that each attachment point on any one aircraft will be excited differently in phase and amplitude from the other attachment points. Further, the nature of attachment-point motion is a function of the interaction of dynamic properties of the local airframe structure and the shelf. The final motion of the shelf of electronic equipment will result from an excitation-level and frequency-dependent force balance at the attachment points. This relative relationship of attachment-point motions will be different for each aircraft because the shelf and various airframes will not interact in an identical manner.

A true equivalence technique would be one which could be used to compare directly the vibratory experience of our example shelf of electronic equipment between aircraft. Further, this true equivalence technique could be used to design a laboratory simulation of the flight environment with such accuracy as to reproduce the complex motion expected in each flight situation. This idealistic goal may be approached for a very simple specimen and environment, such as for a capacitor mounted to the frame of a constant-speed stationary motor. There is serious question, however, whether it is either technically feasible or economically desirable to achieve an absolute equivalence for the more complex specimen and the more complex mounting situations.

The failure of many vibration testing practices and specifications to accommodate the interaction of the tested specimen with the supporting structure, as may be experienced in actual service, can lead to the generation of test results of little value.

One particular weakness of current vibration test practices is the usual requirement that a very stiff test fixture be used to support the tested specimen. As noted by On [89] and Pulgrano [90], the use of a very stiff test fixture can cause great changes in the dynamic response of a specimen. When the specimen is secured to a stiff fixture, the fixture itself will tend to stiffen the specimen and change its vibratory response characteristics, and the stiff fixture is not significantly affected by the dynamic characteristics of a specimen which is mounted on it.

When the specimen is very stiff and its weight is small relative to its supporting structure, the presence or absence of the specimen may not have a significant effect on the vibratory response characteristics of the supporting structure. In these few cases present test practices are probably satisfactory [91].

The test vibration input is usually specified by an envelope of peak sinusoidal acceleration plotted against frequency, or by a PSD-vs-frequency plot for random vibration. The specified test envelope may encompass many sets of data taken by monitoring accelerometers placed at the specimen mounting points during actual service. In this case the test vibration input envelope represents motion resulting from interaction of the specimen and its supporting structure when they are acted upon by a system of dynamic forces. The test specification usually causes the specimen to be mounted to a very stiff fixture which results in a different interaction between the supporting structure and the specimen. Applying sufficient power so that an accelerometer mounted on the fixture will indicate the required test excitation may result in more energy forced into the specimen at its response frequencies than it would experience in service. This approach often leads to overdesign and unnecessary expense in modifying equipment that fails the overly severe test.

The objective of mechanical-impedance equivalence is to duplicate the dynamic structural behavior of a tested specimen when its supporting structure is replaced by a test fixture. The purpose of equivalence is to cause equal damage (wearout or performance malfunction) between normal field use and laboratory dynamic environments. Unfortunately the damage processes are poorly understood. As a result the structural dynamic characteristics of a specimen's mounting structure must be very accurately reproduced in the test laboratory to assure equal damage.

Several techniques have been suggested to improve vibration testing practices by accounting for the interaction between a specimen and its supporting structure. The techniques covered in this chapter include the mechanical-impedance methods of response limiting, input power control, input force control, and vibroacoustic testing.

The primary objective of this chapter is to discuss variables associated with mechanical-impedance equivalence and to review the work of engineers who have attempted to find suitable mechanical-impedance-equivalence testing techniques. No specific method is outstanding or generally accepted. Each should be understood so that the best method may be selected and applied for specific requirements.

5.2 Impedance Equivalences

Several methods have been proposed to reduce discrepancies in the laboratory simulation of field vibration. They involve the alteration of conventional laboratory testing practices to achieve results which more closely duplicate actual service. Approaches covered in this section include response control, input power control, input force control, and vibroacoustic testing.

Prior to a discussion of various approaches to mechanical-impedance equivalence, it may be necessary to review the basic terminology and concepts of mechanical impedance. A review of terminology and concepts adequate for use of this monograph is given in the next section.

Fundamental Concepts

As pointed out by Salter [92], the consideration of motion without the consideration of the originating force may not adequately describe the input or response of a specimen to a vibratory excitation. To do so would be like attempting to understand the performance of an electrical circuit by looking only at voltage and ignoring impedance and current flow.

Mechanical impedance is defined as the ratio of the driving force acting on a specimen to the resulting specimen velocity. If the velocity is measured at the same point i where the force is applied, the ratio is called the driving-point impedance:

$$Z_i = \frac{F_i}{\dot{y}_i}, \tag{5-1}$$

where

Z_i = mechanical driving-point impedance
F_i = the applied force
$\dot{y}_i$ = the velocity at the point of force application.

The terms in Eq. (5-1) are complex numbers.

Mechanical-impedance measurements may be obtained by computing the ratio of the force at a point i to the velocity at some other point j on the specimen,

$$Z_{ij} = \frac{F_i}{\dot{y}_j}, \tag{5-2}$$

where Z_{ij} is called the mechanical transfer impedance.

It is frequently more convenient, especially when deriving an electrical circuit analog of a mechanical system, to work with the inverse of mechanical impedance. The inverse is called mobility and may be either a driving-point mobility or transfer mobility,

$$\bar{M}_i = \frac{\dot{y}_i}{F_i}, \tag{5-3}$$

where $\bar{M}_i$ is the mechanical driving-point mobility. Although mechanical impedance and mobility are defined on a velocity basis, the impedance concept is sometimes extended to include acceleration and displacement responses.

When the ratio of driving force to displacement is used, the result is

$$\bar{D}_i = \frac{F_i}{y_i}, \tag{5-4}$$

where $\bar{D}_i$ is the dynamic modulus at point i, and y_i is the displacement. The reciprocal of the dynamic modulus is called the receptance $\bar{R}_i$. The ratio of the driving force to the acceleration is sometimes called the apparent weight,

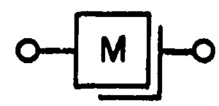
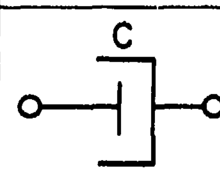
$$\bar{W}_i = \frac{F_i}{\ddot{y}_i}, \tag{5-5}$$

where $\bar{W}_i$ is the apparent weight and $\ddot{y}_i$ is the acceleration of the specimen taken at location i.

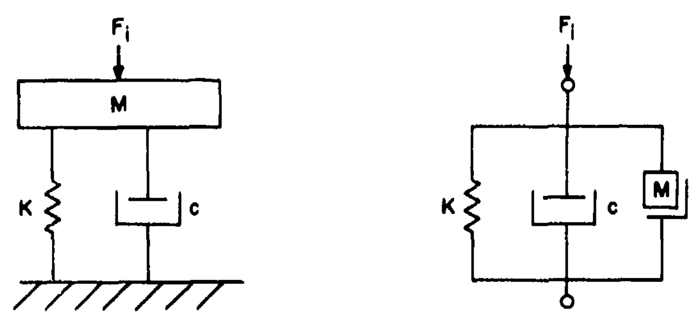
Idealized mechanical system elements with lumped constants may be assembled to form physical systems. These basic elements include the mass, spring, and damper, as shown in Table 5-1. They may be used as building blocks just as resistors, capacitors, and inductors are used in electrical circuits. Consider the simple mechanical system shown in Fig. 5-1a. Its corresponding mobility diagram is shown in Fig. 5-1b. The three elements of the mobility diagram are in parallel, so we have

$$\begin{aligned} Z_i &= Z\,(\text{spring}) + Z\,(\text{damper}) + Z\,(\text{mass}) \\ &= c + j\left(\omega M - \frac{K}{\omega}\right), \end{aligned} \tag{5-6}$$

Table 5-1. Idealized Mechanical Elements

Element	*Symbol*	*Impedance**	*Mobility**	*Force*
Mass		$Z = j\omega M$	$\bar{M} = \frac{1}{j\omega M}$	$F = M\ddot{y}$
Massless Spring		$Z = \frac{K}{j\omega}$	$\bar{M} = \frac{j\omega}{K}$	$F = Ky$
Damper		$Z = c$	$\bar{M} = \frac{1}{c}$	$F = c\dot{y}$

*$j = \sqrt{-1}$



(a) The lumped system (b) Mobility diagram

Fig. 5-1. A simple mechanical system.

where $j = \sqrt{-1}$. Also, in terms of mobility,

$$\frac{1}{\bar{M}_i} = \frac{1}{\bar{M}\,(\text{spring})} + \frac{1}{\bar{M}\,(\text{damper})} + \frac{1}{\bar{M}\,(\text{mass})}$$

$$\bar{M}_i = \frac{c - j\left(\omega M - \dfrac{K}{\omega}\right)}{c^2 + \left(\omega M - \dfrac{K}{\omega}\right)^2} \; . \tag{5-7}$$

Mechanical impedance is a frequency-dependent complex number. It is expressed as the sum of real and imaginary components or as a magnitude $|Z|$ and an angle θ:

$$Z = \mathbf{Re}\,(Z) + \mathbf{Im}\,(jZ) = |Z| \exp\,(j\theta) \ , \tag{5-8}$$

where

$$|Z| = \left\{\left[\mathbf{Re}\,(Z)\right]^2 + \left[\mathbf{Im}\,(Z)\right]^2\right\}^{0.5} \tag{5-9}$$

and

$$\theta = \tan^{-1} \frac{\mathbf{Im}\,(Z)}{\mathbf{Re}\,(Z)} \ . \tag{5-10}$$

It is sometimes convenient to plot impedance magnitude vs frequency and phase angle vs frequency. Such a plot illustrates that the impedance of a damper is a constant at all frequencies, the impedance of a mass has a slope of +1 and crosses the $f = 1$ line at $Z = 2\pi M$, and the impedance of a spring has a slope of -1 and crosses the $f = 1$ line at $Z = K/2\pi$.

An impedance plot for the simple example of Fig. 5-1 is shown as Fig. 5-2. The impedances of the ideal mechanical elements are simple straight lines because they are either real or imaginary. By inspection of Fig. 5-2 we see at low frequencies that the system impedance approaches spring impedance and the system is called stiffness controlled; at high frequencies the system impedance approaches mass impedance and the system is called mass controlled.

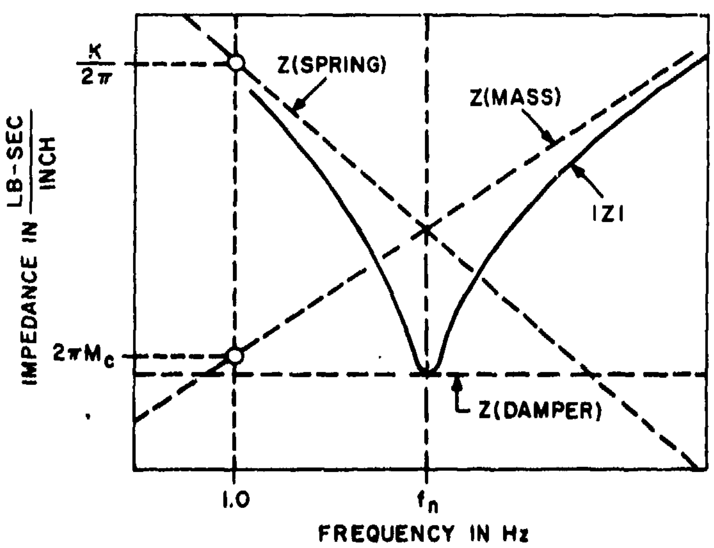


Fig. 5-2. A mechanical impedance plot

Mechanical-impedance measurements may be taken on actual specimens by several methods. In the most common a variable-frequency force generator is used to excite a specimen, and corresponding velocity measurements are taken at the point of force application (for driving-point impedances) or at a remote position (transfer-point impedance). Because most real specimens are not as simple as our three-element example, typical mechanical-impedance plots are usually more complex, as illustrated by Fig. 5-3. In Fig. 5-3 points a, c, and e represent significant resonances, and points b, d, and f are "antiresonance" points.

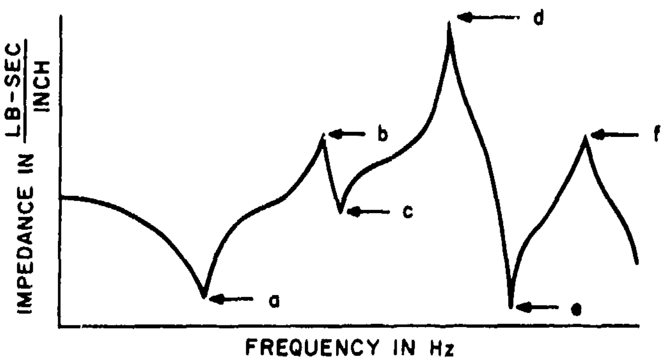


Fig. 5-3. A typical point-impedance plot.

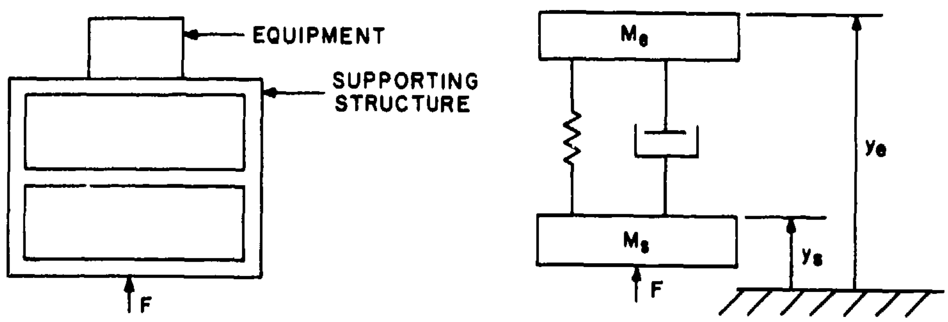
An antiresonance frequency is a frequency at which the impedance magnitude reaches a maximum value. The antiresonant situation is easily visualized by considering a simple series spring-mass system. The maximum mechanical-impedance frequency or antiresonant frequency of the simple spring-mass system is the frequency where the spring-mass system acts as a classical dynamic vibration absorber [74].

For a more detailed review of the fundamentals of mechanical impedance the reader may consult [49,92,93].

An Example

Before discussing laboratory techniques which have been used to achieve impedance equivalence, we shall consider an example of how current testing practices can cause a lack of similarity between laboratory and field failures. The lack of test equivalence which can occur was clearly pointed out by Blake [94], Salter [95], Vigness [96], and others such as Pulgrano [90], Silver [97], and Painter [98]. Each of these authors concluded that specimens in general are severely overtested at the frequencies most damaging to them by the practice of enveloping field data and forcing that envelope to appear at specimen mounting points.

Figure 5-4a shows a piece of equipment mounted to a supporting structure which is excited by a sinusoidal force. A two-degree-of-freedom model of the system is given in Fig. 4-5b where the equipment mass is coupled by a parallel spring and damper to the mass of the supporting structure. For some set of system parameter values the response of the system to a sinusoidal sweep excitation may be as shown in Fig. 5-5. Assume that someone has measured the support response ($\ddot{y}_s$) of many similar mechanical systems which have different values of mass, stiffness, and damping. The result of these measurements, when plotted against frequency, might appear as in Fig. 5-6. An envelope curve, as shown, could be found which encloses say 95% of the data points. This envelope curve, in turn, is applied as a vibration specification on fixture motion for the equipment.



(a) The system (b) Mechanical model

Fig. 5-4. An example system.

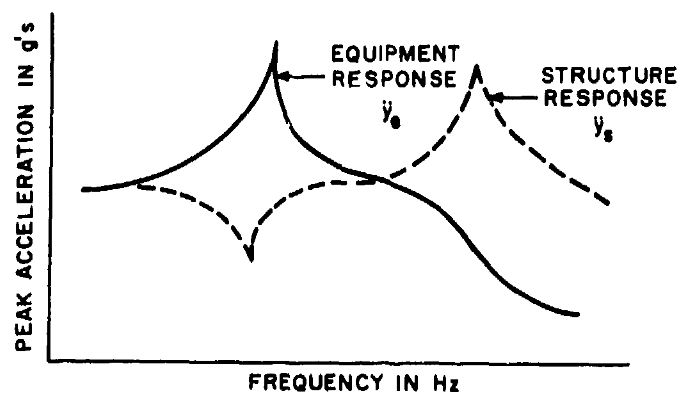


Fig. 5-5. Response of the example system (Fig. 5-4) to a sinusoidal sweep.

The result of testing the equipment portion of the system to the envelope curve is shown in Fig. 5-7. In this simplified example it can be seen that the response of the equipment ($\ddot{y}_e$) to the specified laboratory test as defined by the envelope of Fig. 5-6 is very different from the original situation where the equipment response was allowed to be modified by the structure. Point *a* of Fig. 5-7 is an antiresonant frequency of the supporting structure (note that positive peaks in Fig. 5-3 were called antiresonant peaks when plotting impedance against frequency) which resulted from the interaction of the equipment and the

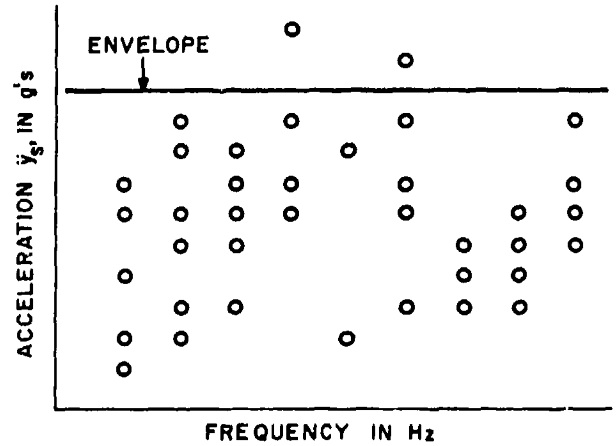


Fig. 5-6. A plot of example field data.

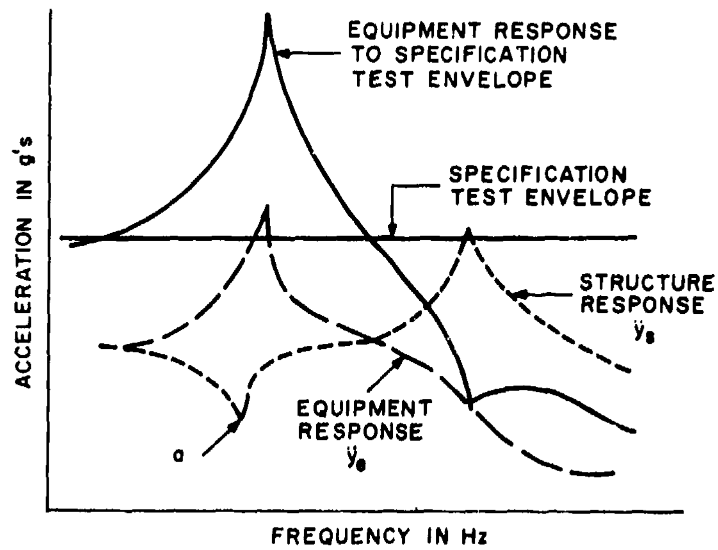


Fig. 5-7. A comparison of the example system response to a test envelope and normal response.

structure. The imposed test envelope disallows the occurrence of the structural antiresonance, thereby causing more energy to be forced into the equipment at its resonance frequency than the energy which would be available in actual service.

Response Control

Response control techniques improve the simulation of mounting impedance by controlling the laboratory test using specimen responses rather than specimen input excitations.

A simple technique described by Curtis and Herrera [99] consists of "notching" the input spectrum. To "notch" the input spectrum means to reduce the test input acceleration levels in selected frequency bands. The notched frequency bands are those where specimen responses might exceed the levels which would be reasonably expected if the specimen were mounted to its normal supporting structure. The notched bands correspond to antiresonances and reflect the inability of the in-service supporting structure to drive the specimen. This approach has an inherent problem in that the environmental engineer must find the location of the critical specimen responses and select correspondingly appropriate response levels.

Another response control technique was suggested by Vet [100]. It involves the use of a suitable envelope of the ratio of test item response to test input vibration level. In his study Vet constructed a multi-degree-of-freedom analog computer model using frequencies, damping, weights, and stiffnesses comparable to the values expected in the case of an item of electronic equipment attached to a missile or aircraft structure. He was unable to find any combination of parameters which yielded resonant amplifications exceeding 1.5:1. The ratio of 1.5:1 was also suggested by Salter [101] to describe a suitable response envelope.

The latter approach suggests that a vibration test may be determined which recognizes mechanical impedance by limiting the input excitation amplitude (i.e., test fixture responses) so that critical specimen responses will not exceed some "amplification factor" times the originally specified input test amplitude. That is,

$$V_{\text{input}} = [AF]\, V_{\text{spec}} \tag{5-11}$$

where

V_{input} = the actual test level input to the specimen as a function of frequency

AF = an analytically or experimentally determined amplification factor, which may be frequency dependent

V_{spec} = the specification input test level as a function of frequency.

An illustration of the effect of vibration test response limiting is provided by considering the example system described in the previous section. Figure 5-7 may be replotted under the condition that the input acceleration will be controlled to yield equipment responses not to exceed 1.5 times the specification input acceleration level. The resultant plot is given as Fig. 5-8.

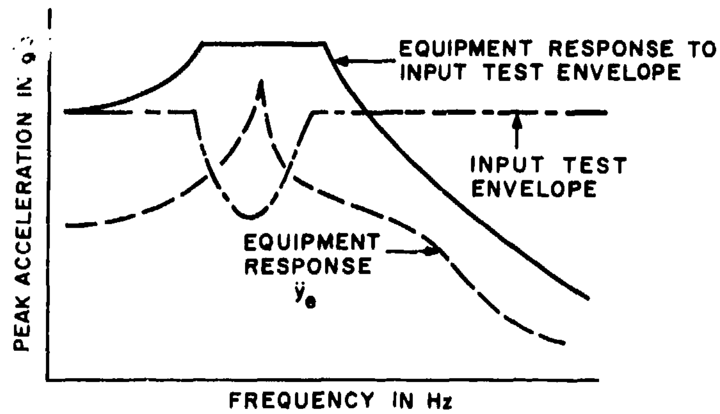


Fig. 5-8. The effect of response-limit testing of the example system (compare with Fig. 5-7).

In addition to the problem of specimen response level and location selection, test repeatability may suffer because minor assembly, structural, or design differences may change the specimen's dynamic characteristics at the immediate mounting of the response-monitoring accelerometer. The drawbacks to response limiting greatly limit its usefulness as a general purpose test procedure.

A change of emphasis is needed in those cases where response limiting techniques are applicable. Field test data should be taken to define equipment responses, rather than trying to determine an envelope of suitable input test levels.

Input Power Control

Input-power-control mechanical-impedance equivalence techniques involve the control of input power to the vibration exciter. This approach may be used directly or in combination with the response-limiting techniques described in the previous section. It is based on the idea that an actual service vibration source has a limited amount of energy available with which to excite a mechanical system.

A mechanical system usually consists of a number of levels, or orders, of structure. A level, or order, of a mechanical system is that portion of a system which can be identified as a single region in an overall model of the system. For example, aerodynamic turbulence excites the skin of a missile (first level), which drives the internal structure of the missile (second level), which carries an equipment mounting bracket (third level), to which is attached the case of an instrument (fourth level), which supports a module chassis (fifth level), and the module chassis is the mounting for a small component part (sixth level).

Available excitation energy originates primarily from external stimuli and not from dynamic properties of the mechanical system. The addition or removal of relatively small equipment or components on secondary or higher order system elements will not affect the availability of energy. Due to the limited amount of excitation energy available, the response of any level of structure is a function of the response of both higher and lower orders of structure. That is, when one order of structure exhibits an antiresonance, the next lower level will experience small responses. When the higher order structure exhibits a resonance, the lower level will not be as constrained and will experience larger responses. The same effect is attempted in the laboratory by the technique of input power control.

Input power control is achieved experimentally by mounting a "dead mass" on the laboratory vibration test fixture and plotting the exciter input power as a function of frequency when the fixture is driven to some specified acceleration level. A dead mass has the same weight (mass) as the test specimen; however, it does not have significant resonant or antiresonant responses in the range of test frequencies. The final test consists of vibrating the test specimen using the experimentally derived power-vs-frequency curve to control the excitation source.

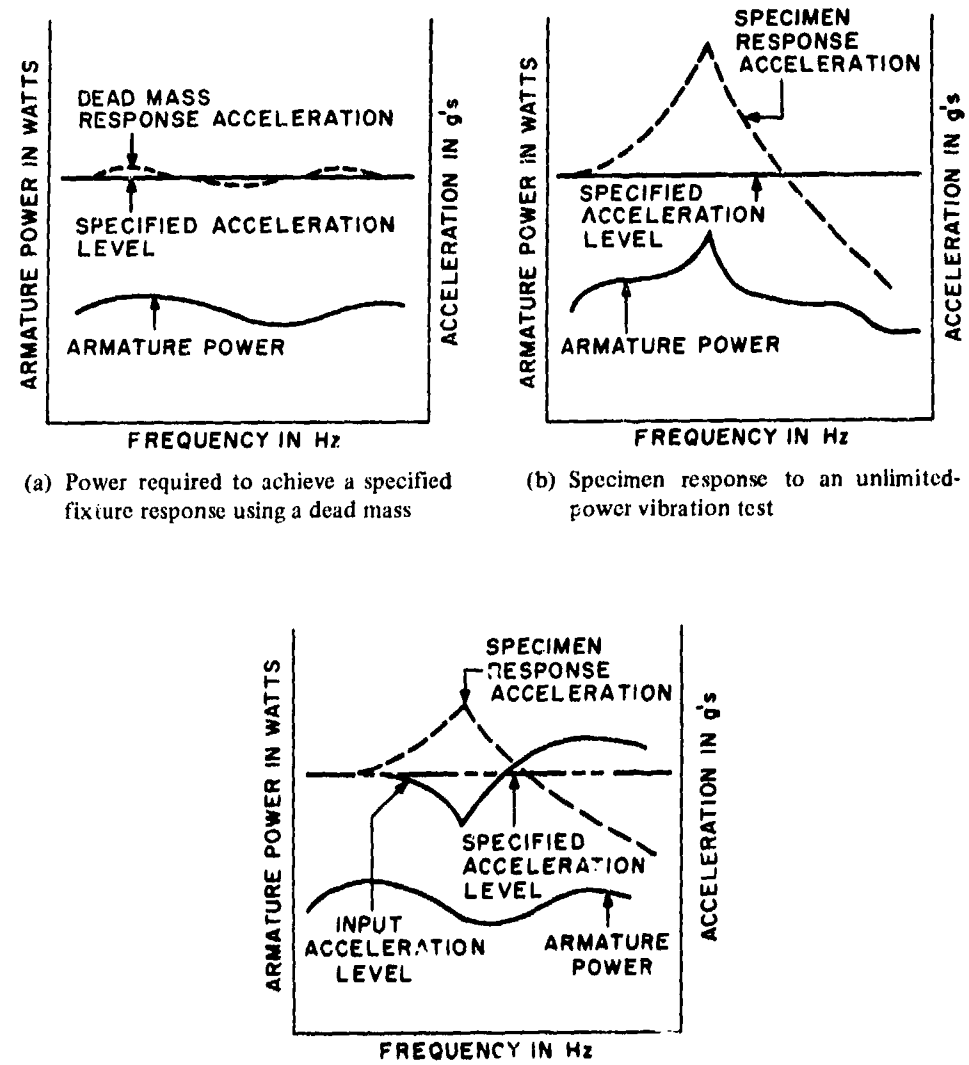
Figure 5-9a shows the exciter armature power required to cause the vibration test fixture, when loaded by a dead mass, to respond to a specification input acceleration level. Figure 5-9b shows how the exciter input power requirements increase at specimen resonance if the vibration test fixture, when carrying the specimen instead of the dead mass, is forced to respond to the same specification input acceleration level. Figure 5-9c shows the response of the same hypothetical specimen when the input power is controlled. Note that the vibration-fixture acceleration level drops realistically at the specimen resonance, and the equipment response is also lower at resonance.

One problem with input power control testing is that the specimen will be overtested at frequencies where it exhibits a low mechanical impedance. This tendency can be partially eliminated by controlling input acceleration levels at the fixture in addition to controlling input power to the exciter. The input acceleration level at the fixture would be limited to the specification level at those frequencies where there is a tendency to overtest. The effect of dual control is illustrated by Fig. 5-10.

Input Force Control

The technique of achieving a mechanical-impedance equivalence by input force control is similar to the input power control method except that force is substituted for power. If the force transmitted by the specimen's field-supporting structure were known and if the force transmitted to the specimen by the laboratory excitation source could be measured, it would be a simple matter to control the excitation source to provide a laboratory test where the dynamic loads closely approximated those applied in the field environment.

The techniques of force control testing have been investigated by Painter [98], Otts [102], Salter [101], Murfin [104], Belsheim and Harris [105], and Ballard et al. [106], among others.



(a) Power required to achieve a specified fixture response using a dead mass

(b) Specimen response to an unlimited-power vibration test

(c) Specimen response to an input-power-controlled test

Fig. 5-9. An example of input power limiting.

Painter suggested that both force and acceleration data be gathered in the field and used for the control of laboratory tests. The laboratory excitation source should not be allowed to exceed either the acceleration or force-field envelopes. Another approach would employ input force control and also use the test specification to determine acceleration response limits for the test fixture. The result would be a test which is force control limited at specimen resonances and acceleration limited at other frequencies. Either of these approaches would be

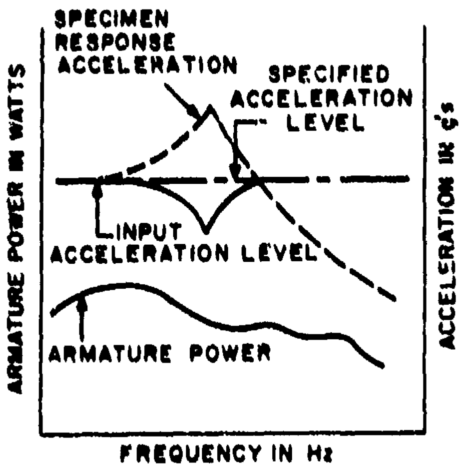


Fig. 5-10. An example of input power and acceleration control.

satisfactory when the tested specimen and supporting structure have defined and relatively invariant dynamic properties. The measured force envelope is a function of the combined dynamic properties of the supporting structure and the equipment [105]. When dynamic properties change, as when an item of equipment is used in several different vehicles, the measured force envelope will also change.

Otts [102] considered the use of an effective foundation mass located between the test specimen and the excitation force. The advantages of this approach are that the specimen's response will influence the response of the foundation, and the response of the system is not affected by the armature characteristics of the vibration exciter. A simple system representation is shown in Fig. 5-11.

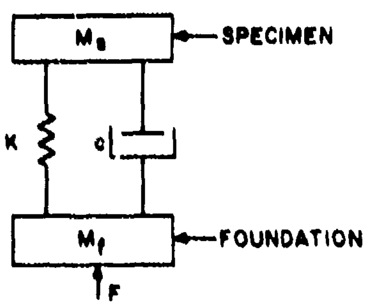


Fig. 5-11. A simplified mechanical system.

For his work on a Scout rocket, Ott used an intermediate structure to simulate the mass, spring rate, and damping characteristics of the actual specimen supporting structure. Test data revealed that at low frequencies, below 120 Hz, the test was accurate, with nonrepresentative responses occurring at foundation antiresonances and frequencies above 200 Hz. The result of this work illustrates that specimen responses are influenced by foundation dynamic characteristics which in turn must be accurately reproduced to assure a valid test.

Another approach would be to duplicate the specimen mounting structure dynamics in the laboratory, using a multimodal fixture such as reported by Scharton [107]; however, the development of these structures can be costly and time consuming.

Another approach using mechanical-impedance theory and the usual field acceleration data to compute the test input force envelope was suggested by Murfin [104]. This approach involves three steps:

1. Experimentally determine the specimen's apparent weight by performing a sinusoidal sweep test in the laboratory and measuring the applied force and the specimen mounting-point acceleration history.

2. Use an envelope of field input acceleration data to compute the test force:

$$F_i = \bar{W}_i \ddot{y}_i \ , \qquad (5\text{-}12)$$

where $\bar{W}_f$ was determined in Step 1, and $\ddot{y}_f$ is the field input vibration envelope.

3. Perform the final test using the computed force of Step 2 as a specimen input, and limit the specimen input acceleration to the field acceleration data envelope.

Murfin's approach negates the need for a foundation mass in the test; however, the effects of field foundation mass will influence the field vibration data used in Steps 2 and 3.

Input force control techniques also suffer from drawbacks similar to other mechanical-impedance techniques. One inherent assumption implicit in measuring or controlling force or acceleration is that the measured (controlled) quantity is taken at a single, well-defined point. Unfortunately, most specimens have two or more mounting points which, if a very accurate test were required, would need to be controlled individually yet produce proper magnitude and phase relationships.

Vibroacoustic Testing

A recent testing technique employs large acoustic chambers to simulate acoustic excitation levels on structures or portions of a structure. Vibroacoustic testing is a test method used to achieve realism rather than an interaction equivalence. However, the objective of vibroacoustic testing is to provide a method of structural excitation as complex as that which would be experienced in service. Acoustic excitation of a complete vehicle such as a spacecraft can yield an accurate duplication of mechanical impedances at individual equipment locations. A listing of current references on the subject of acoustic testing is provided in Table 5-2. The term *vibroacoustic* was used to introduce this paragraph because in various instances the acoustically tested structure is simultaneously excited by one or more directly coupled sources in addition to the acoustic noise.

In some cases the use of acoustic facilities was suggested for the purpose of exciting structures which are normally excited by mechanical means. Scharton [108] cautions that the inefficiency of acoustic excitation relative to mechanical excitation warrants serious consideration.

Table 5-2. Acoustic Testing References

Author	*Reference*	*Date*
Arcas	109	1969
Kaplan	110, 111	1969
Elsen	112	1968
Maurer	113	1968
Peverley	114	1968
Schurton and Yang	115	1968
Wren, et al.	116	1968
West, et al.	117	1968
Bost	118	1967
Lyon	119	1967
Mueller and Edge	120	1965
Putukian	121	1965
Eldred	122	1964
Noiseux	123	1964
McGowan and Frasca	124	1963
Mahaffey and Smith	125	1960

CHAPTER 6
VIBRATION TESTING APPLICATION

6.1 Preliminary Considerations

It is not difficult to define a general criteria for a good test, but it is difficult to implement such a test. A good test will create a failure in those equipment items which fail in service, with failures in both cases being similar. Conversely, a good test will not produce a failure of those equipments which survive in service. Based on these criteria, a good laboratory test must meet several requirements. The task of establishing a test for one equipment used in one application varies significantly from the task of creating a general purpose test specification. Plunkett [126] suggested that environmental test specifications should be tailored to specific applications because a standardized specification procurement cannot fulfill all requirements resulting from the variety of eventualities which arise. Silver [97] noted that the currently used general purpose environmental specifications are rarely representative of actual service conditions because arbitrary levels are utilized to achieve reproducibility and standardization.

From a specification writer's viewpoint there are several important considerations which must be accommodated by a general purpose specification: (a) it is necessary to specify vibration requirements in a common form which will allow comparison among similar designs, (b) it is necessary to have a simple and clearly worded criteria by which it is possible to define product quality requirements contractually, (c) product design criteria often must be defined before a service environment can be defined, (d) the product which successfully meets specified criteria must perform under several service conditions when attached to various structures, and (e) it is necessary to define specification compliance in terms such that inspectors can determine whether or not a product meets the specified conditions.

Harvey [127] pointed out that because of the nature of current general purpose specifications, in many instances hardware is not designed to meet its service environment but rather to pass a conservative test established to simulate the environment. Unnecessary conservatism in establishing test margins can result in product redesign and increased test costs which are out of proportion to the value of the test. Othmer [128] observed that not all failures resulting from a general test specification are significant. Redesign is not warranted until applicability of the specification is reviewed, test techniques are investigated, and the specimen is analyzed. Many variables must be considered in the establishment of valid test criteria. It is not a purpose of this monograph to discredit general purpose specifications. They are necessary, but they have weaknesses which should be discussed. Because of these weaknesses, the general purpose vibration specifications should be reviewed to assure that test levels are not established which will lead to unnecessary cost or design change.

The technical and procedural aspects of vibration test selection and the performance of vibration tests have been carefully documented in another monograph of this series by Curtis, Tinling, and Abstein [53]. Duplication of the material they have presented will be avoided here to the largest extent possible; however, it will be necessary to discuss the interrelationship of vibration equivalences and test selection.

6.2 Retention of Realism

Vibration testing, a term which is used regularly in this monograph, should not be interpreted to imply that the vibration equivalences are of concern almost entirely in the laboratory. A major need and use of vibration equivalence techniques is for the translation of a field environment into design and test requirements.

The selection of a proper vibration equivalence is situation dependent, and many variables must be considered. Not only are technical aspects to be satisfied, but also the program or project phase must be considered, and likewise the objective or reason for forming an equivalence.

There are many correct approaches to defining an environmental situation which involves vibratory excitation. The nature of the information available and objectives of the involved environmental engineer usually lead to the selection of the best approach for each situation. For example, the environmental engineer who has overall responsibility for a product will be concerned with the control of design and development activities such that the product will meet certain performance goals. He should have access to relevant service history information. In addition, he must define the desired service requirements. His goal is one of defining a set of design and test specifications which will, when followed by individual product design groups, lead to the development of a product (system) which will meet the desired performance requirements.

A product design engineer may be responsible for using a given set of design and test specifications and designing a product which will meet those specifications. His view of the overall project may not be comprehensive; however, he is intimately familiar with the performance characteristics of his type of product. In some instances he may observe that minor modifications of the environmental test specifications could lead to significant savings in time and cost relative to the design of his product, and would not necessarily compromise the intent of the specification. In those cases it is important that he be able to communicate the changes and concepts to the originator of the specified requirements so that, where possible, the savings may be implemented.

The product design engineer may be faced with another problem, that of attempting to determine whether an existing product will perform in a satisfactory manner when exposed to a new set of vibratory conditions. It is important in this instance that he understands the potential failure mechanisms of his product so that an appropriate set of equivalence techniques may be selected and applied to simulate the changed environmental condition.

A test engineer may have access only to the product and a set of specifications, and may be faced with the task of designing a test fixture and then testing to meet the technical requirements as delineated in the specifications. If the test engineer is neither the system environmental engineer nor the product design engineer, then he has little choice other than to follow the exact word of the test specifications. Such a situation places a large burden on the specifications and does not allow the test engineer the freedom he may need to adjust the specified test requirements to match the capabilities of his test equipment. In addition, the critical details associated with test realism, i.e. mounting and fixture design, may be implemented without an appreciation of the actual service conditions. As a result of these factors it is important that the system and design engineers, as applicable, maintain continuous contact with the progress of each product through all test phases.

6.3 Equivalence Technique Selection

A diagram was prepared to illustrate the interrelationships among the many events involved in vibration simulation and testing. This diagram, Fig. 6-1, will be used as a basis for the following discussion of equivalence technique selection and application. The discussion will be maintained in general terms for the purpose of preserving applicability to a wide range of products and environmental situations. Hopefully, the discussion will be sufficiently specific to guide the reader to the solution of individual problems which involve the application of equivalence technology.

Each use of vibration equivalences is based on common theory, yet in each case the objective for forming an equivalence is quite different.

Definition of the Service Environment

In many instances an accurate definition of a service environment is difficult, primarily because there exist sources of excitation which cannot be either foreseen or controlled by the environmental engineer. These excitation sources, depending upon the specimen under consideration, may include a combination of sources, such as handling, transportation, acoustic excitation, gunfire, aerodynamic buffeting, rotating machinery, propulsion unit motion, or dynamic imbalance. Regardless of the source of excitation it is necessary to characterize the service environment in some manner to allow a rational evaluation of the environment and the probable reaction of specimens exposed to that environment.

The usual measured service data available are motion data; perhaps the motions of some element of the excitation source, or the motions of some element

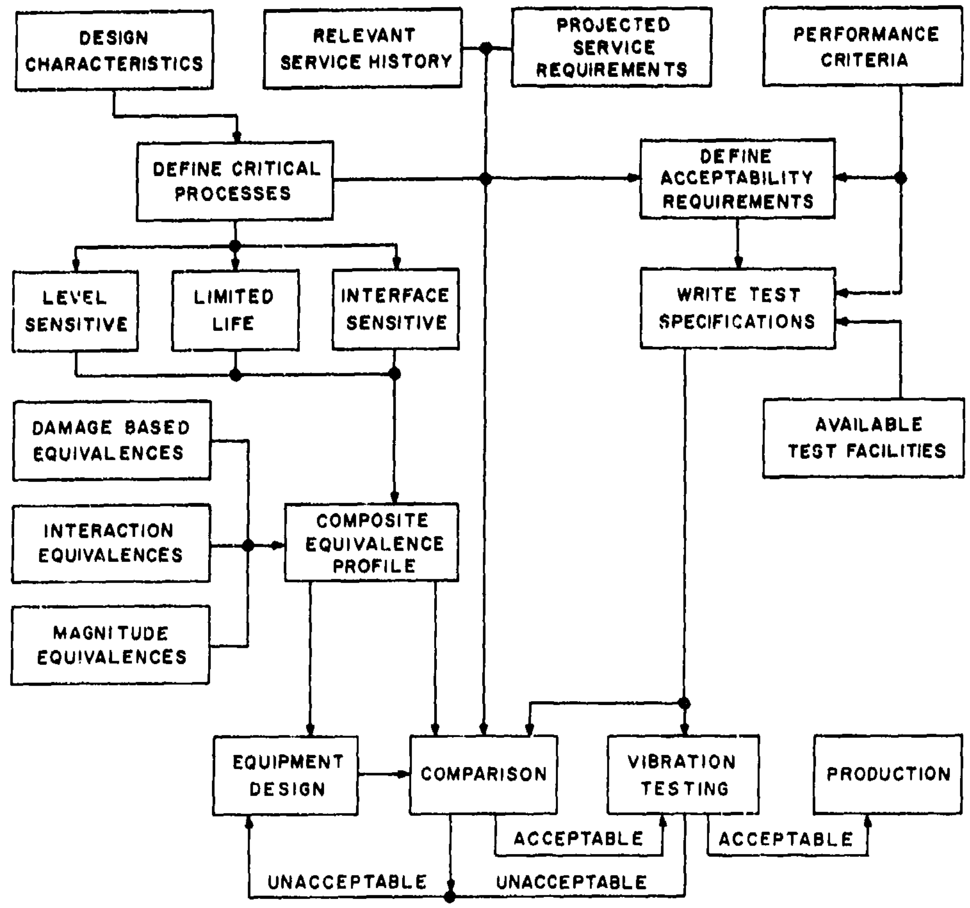


Fig. 6-1. Equivalence technique application.

of the specimen, or both. For practical purposes the source motion is sometimes taken as the local motion of the structure which supports the specimen. Such measurements are valuable because they may be categorized by frequency content as a function of time, and excursion as a function of frequency. Such measurements may easily lead the unwary to believe that equipment response is totally defined by motion data. As an illustration, consider the following statement: "The specimen will perform satisfactorily in vehicle B because it performed satisfactorily in vehicle A, and measured data from vehicle A indicate that the supporting structure was developing acceleration amplitudes equal or above those expected in vehicle B." The rationale may be valid if each of the following three questions will receive an affirmative answer: Was the specimen's response independent of the local stiffness of vehicle A? Will vehicle A and vehicle B be expected to respond to service environments in an identical manner at the mounting location of the specimen? Will vehicle B be subjected to the same service conditions as vehicle A?

An affirmative answer to the first question would mean that the specimen is of sufficient rigidity that it will respond as a unit to any combination of excitations, varying in amplitude and phase, at the attachment points of the specimen to the supporting structure. Such a situation may be approximated by a very rigid specimen mounted to a very flexible structure.

The second question could be expanded to inquire: When the same specimen is mounted in both vehicles, will each vehicular structure respond in an identical manner at the location of the specimen? Again, the point of the question involves the interaction of the structure of the specimen with the structure of the vehicle.

The final question is merely an inquiry as to whether the measured service data taken from vehicle A are an adequate representation of the vehicle A service history, and whether the assumed (or projected) model of the anticipated vehicle B service is accurate. If, as usual, some doubt exists, the environmental engineer should weigh the probability of service failure due to vibratory excitation against the value of service success. Such a process of evaluation may be only poorly quantized; however, this is the process used to establish design and test margins based on judgment, and is a very important facet of vibration simulation and test design.

When defining the service environment it is important to use measured specimen-response motion data as an indicator of service history. These motion data are valuable only if the sources of excitation and the mechanical impedance between the specimen and the excitation sources are defined. Further, the reliability of such information requires investigation so that a simulation of each facet of service environment may be duplicated in time and excitation (response) level.

As an aid to visualization, Fig. 6-2 was prepared to illustrate an example of our hypothetical specimen, which was used in an existing vehicle A and will be used in a new vehicle B. In both cases the vehicles will be excited by traversing certain common terrain, source 1, for which recorded data exist for vehicle A. Also, the effects of another source of excitation common to both vehicles, gunfire, is designated as source 2. It is expected that vehicle B will traverse a more rugged terrain, source 3, and it is necessary to estimate the response of the specimen under these circumstances.

The specimen response spectra, 1-A through 3-B, define the resultant motions of the specimen under each circumstance. In two instances, excitation by source 1 and source 2, the prime variable, other than perhaps differences in the duration of excitation, is the difference in structural design between vehicle A and vehicle B. In this case the interaction equivalence techniques of Chapter 5 are applied. The reaction of the specimen to source 3 when mounted into vehicle B, designated as response spectra 3-B, is a projected service environment based on knowledge of the specimen's response on vehicle A and the expected characteristics of vehicle B.

In this manner the service environment, past and projected, is defined. The primary contribution of the vibration equivalences is that they discipline us to

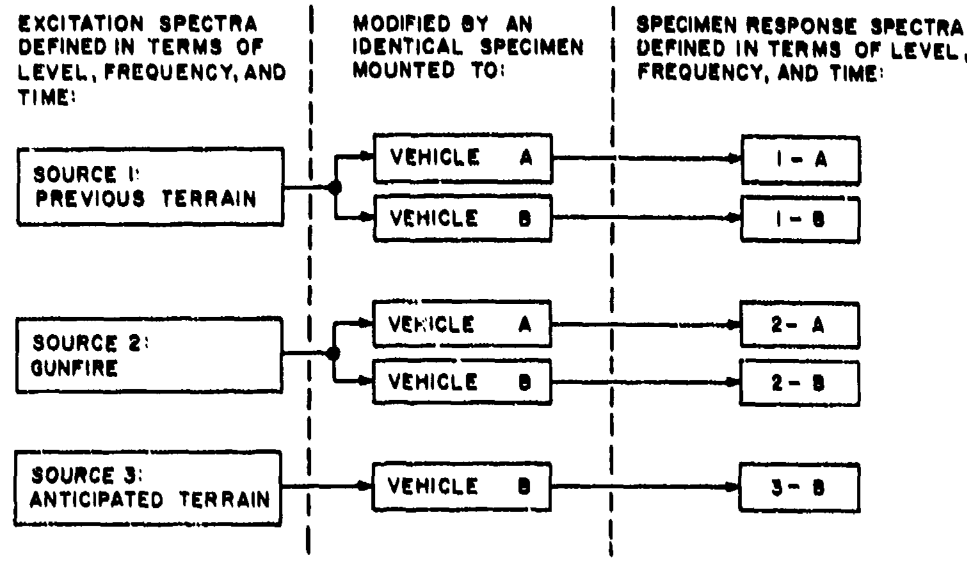


Fig. 6-2. Defining service environments, an example.

define the service environment in terms of the interaction of the specimen with the immediate environment, as well as to define specimen motion in terms of level, frequency, and the duration of exposure at each level.

Representative Service Environments

With the service environments defined, it is possible to combine these environments to form a representative service environment. This representative service environment would be used as a basis for product design computations and the definition of laboratory tests intended to simulate the service environment.

The service environment may be defined as either representative source excitations or as representative response spectra. In either case the interaction relationships between the forcing elements and the responding elements are of interest equal to the motion information. The objective of creating a representative service environment is to establish a situation where a specimen will be allowed to react in a manner which will duplicate the important characteristics of specimen reaction under service-induced excitation.

In most instances the excitation source for a specimen is the response of the structure supporting the specimen when that structure is subjected to environmental stimuli. The supporting structure will usually amplify the excitation energy at certain frequencies and attenuate the excitation energy at other frequencies. In addition, for a given specimen and supporting structure configuration, the duration of exposure at different excitation levels at each response frequency will provide an additional variable.

Thus it may be seen that the reduction of several service environments to one or a few representative environments will require the creation of new excitation (response) spectra, scaled in level at selected frequencies, and described by the duration of exposure at each frequency.

Interpretation of Acceptability Requirements

Acceptability requirements are those standards applied to a given specimen in terms of acceptable specimen performance under a given set of service conditions. Acceptability requirements appear in equipment specifications frequently as specimen operational performance standards during or following the imposition of a vibration test which was designed to represent the service environment. Acceptability requirements (recall Fig. 6-1) are defined by the general performance requirements of the system of which the specimen is an element. These general performance criteria are restated in terms of the specific functions performed by the specimen, and appear in the specimen's test specification.

After the acceptability requirements have been established, the vibration-equivalence techniques are used to interpret these requirements in terms of specimen design criteria. This interpretation involves a reduction procedure which has as a goal the creation of a composite equivalence profile for the specimen. Once a reasonable composite equivalence profile has been formed, it is possible to estimate the effects of various design concepts and variances in the projected service requirements or testing requirements.

An early step in the formulation of a composite equivalence profile is to identify the various critical processes which may lead to a failure of the specimen to meet the acceptability requirements. Once these critical processes are identified and categorized as level sensitive, limited life, or interaction sensitive, they may be individually modeled by the techniques described in Chapters 3, 4, and 5. The resultant equivalence profile may become quite complex because different failure processes may predominate at different frequencies, and the predominant type of failure process at any one given frequency may change with excitation level. Although the potential for significant complexity may appear to be a deterrent to the formulation of a composite equivalence profile, this complexity is often unwarranted. For example, it is often discovered for a given specimen that only one or two failure processes are the limiting processes which must be modeled. Alternatively, it may be discovered that the service vibration spectra are simple and very similar at the response frequencies of the specimen. Also, a lightweight, rigid specimen may be involved which is not greatly influenced by the design of the exciting mounting structures to which it is mounted. Any of these situations will greatly reduce the complexity of the composite equivalence profile.

Application of the Composite Equivalence Profile

The results of creating a composite equivalence profile are of importance in equipment design because the most probable failure modes have been defined and the designer will know where to concentrate his efforts. Further, the equivalence techniques may be used to compare the effects of various service conditions by a comparison of the estimated specimen response to those conditions.

The composite equivalence profile is applied in testing as the basis for test-time and test-type scaling, in the prediction of specimen performance under different test conditions, and as a diagnostic aid in the event of a specimen failure.

A final and important use of the vibration equivalence techniques is in the area of quality control, where simple yet meaningful specimen performance tests may be designed and run to gain prior assurance that the specimen will perform properly under service conditions.

6.4 Equivalences and Testing

Test Accuracy

There are several recognized problems which influence the degree to which a test is an adequate representation of an actual or projected service vibration environment.

A primary area of concern is caused by the tendency of many engineers to place a great deal of importance on data gathered from the test of a single specimen. The data from a single engineering prototype or a first production item, for example, are taken to represent the ability of all like items to pass certain vibration test requirements. As observed by Small [129], a main purpose of laboratory vibration testing is to ensure adequate hardware reliability relative to structural integrity and acceptable functional performance. The problem here is that the testing of one specimen does not necessarily constitute an adequate sample size to assure reliable service performance of like specimens. Figure 6-3 shows a simplified interrelationship between sample size, reliability, and confidence level. For example, 30 samples of a specimen must pass a given test without failure to achieve a 60% confidence that the tested specimen will have a reliability of 0.97 under the tested conditions.

It is desirable to have a large number of test specimens so that incremental stress level data can be gathered to describe a distribution of strength for the tested item. Once this distribution and a distribution of vibration levels are available, one can predict with a high degree of confidence the percentage of failures expected in service. Large sample sizes are practical and commonly used for small low-cost component parts, but prohibitive costs are involved when attempting to apply lot-sampling techniques to high-cost and low-production-volume products. The result is a forced compromise because test data are necessary to show design adequacy even if that data are indicative rather than an accurate representation of product reliability.

Problems with service predictions arise because service data seldom exist at every desired location and for every type of service history. Those vibration tests where data are available were probably conducted on a limited number of

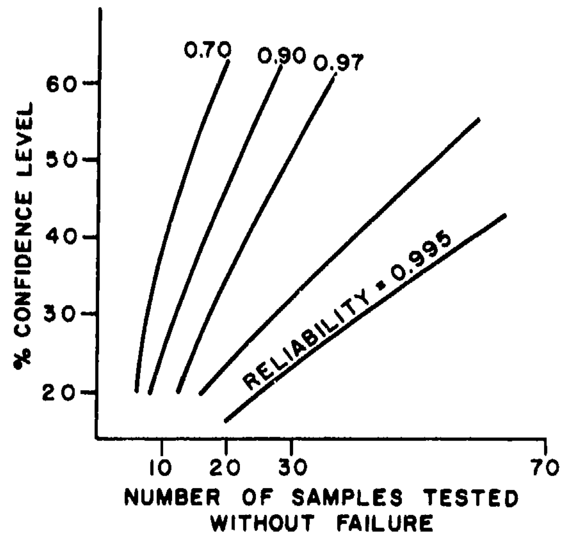


Fig. 6-3. Sample size vs confidence level [129].

test items, frequently on a success/failure basis, and yield very little information about distributions of strength. Some significant vibration histories are not stationary processes and usual data reduction techniques are not adequate to describe these processes. Also, test items are not usually subjected to three axes of motion simultaneously, which occurs in service.

The practice of enveloping sets of predicted or measured structural response data and using these envelope spectra as test excitation spectra is conservative. This technique will eliminate [98,101,130] response valleys which may represent an interaction of the tested item with the mounting structure. Such an approach ignores the fact that at certain frequencies the test item will tend to act as a classical vibration absorber. At these frequencies, the test machine must provide unrealistically high excitation forces to maintain the envelope vibration levels.

One approach used to account for uncertainties and data scatter is to apply a test level factor, or test margin, which is intended to lead to a calculated amount of overtesting. While the selection of this factor is largely a matter of experience and engineering judgment, the factors used for time-scale changes should be based on cumulative-damage equivalence theory to avoid severe overtesting.

In addition to the above noted sources of potential inaccuracy in vibration testing, several other errors may be introduced, such as minor assembly and design variances of the specimen, accelerometer positioning, and erroneous test execution and data reduction procedures. Many of these areas of potential inaccuracy appear to be ignored to varying degrees by current testing practices. The simplifying assumptions which have been used to form the vibration equivalence techniques are also sources of potential inaccuracy. However, the known

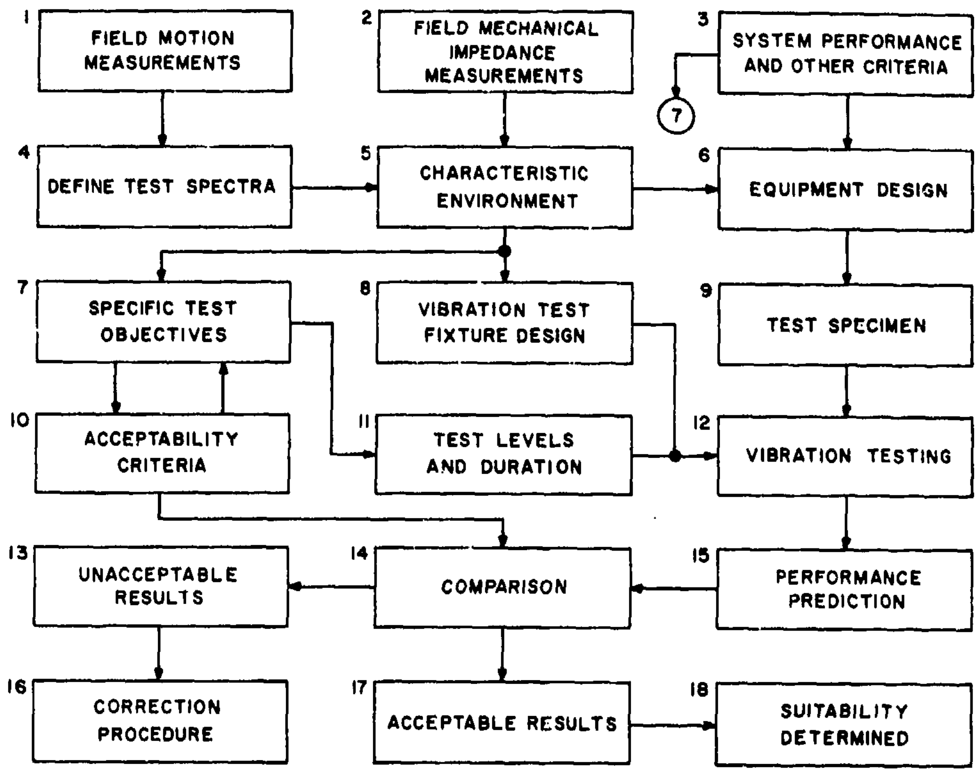


Fig. 6-4. Vibration-test event diagram.

Equivalence Category		Process†	Application
Damage Based		1 → 4	Combine a multiplicity of data into representative spectra and data
		3 → 6	Define acceptability and failure criteria on a functional basis
		5 → 6	Transform representative spectra and data into general design and test criteria
		3, 5 → 7 ⇄ 10	For a given set of test objectives, define specific test spectra and acceptability criteria
		7 → 11	Transform (if required) test spectra into form matching test facility capabilities
		15 → 14	Compare test results with acceptance criteria on equal basis
Interaction		2 → 5	Define a standard or set of standard interaction criteria to correspond with process 1 → 4
		3 → 6	Project future applications into design requirements
		5 → 6	Interpretation of interaction standards for design criteria
		5 → 8	Interpretation of interaction standards into fixture hardware
		8 → 12	Fabrication and proof-of-test-fixture adequacy
		12 → 15	Definition of specimen performance under specific interaction conditions
		15 → 14	Assure that comparisons are made on like interaction basis

†A process is the activity of moving from one event to the next, i.e., 1 → 4 means to move from Event 1 to Event 4. See Fig. 6-4 for a definition of events.

Fig. 6-5. Application of equivalence techniques to event diagram processes.

inaccuracies introduced by the careful application of the vibration-equivalence techniques should lead to less variability in test results than the unknown inaccuracies now accepted in current testing practice.

Test Applications

A vibration test may be conducted for one or more of these purposes: (a) to obtain necessary information for equipment design, (b) to evaluate the suitability of a final design, (c) to demonstrate the capability and quality of a design, or (d) to assure that initial product quality has been maintained during the production of an approved design. Each requires individual consideration and may be viewed differently by a customer (who originates a test requirement) and a supplier (who must show that his product will meet the test requirement). There are other factors which will influence the vibration test: (a) the level of assembly which is to be tested, from the component through the unit, subassembly, assembly, and overall system; (b) the nature of the acceptability or failure criteria; and (c) the economic factors such as test and specimen cost, schedule, and the cost of a failure-retest cycle. Each of these factors must be considered in the design of a test to achieve the best balance for each test situation. Most of these factors have been discussed earlier in this monograph or are factors which can be most accurately evaluated by the individual who must design a test.

Regardless of the immediate factors which influence vibration test specifications and procedures, there is a general pattern of events associated with vibration testing. Environmental engineers associated with vibration testing should be aware of the existence of each event even though their immediate needs may appear to encompass only a portion of the whole. The general pattern of events associated with vibration testing is depicted by Fig. 6-4. This figure may be used as a planning aid by considering each event in relationship to a specific project.

A summary of the application of the vibration equivalence techniques to the events of Fig. 6-4 is given as Fig. 6-5. The latter figure discerns between interaction equivalence applications and damage-based equivalence applications. Those processes which are not listed in Fig. 6-5 may also involve an application of equivalence techniques by implication.

Examination of Fig. 6-4, coupled with the information of Fig. 6-5, graphically illustrates the relationship of the vibration equivalence techniques to the entire area of vibration simulation and testing.

CHAPTER 7
CONCLUSIONS

The vibration-equivalence techniques have been shown in relationship to the entire area of vibration simulation and testing. The factors considered included field data, projected service requirements, acceptability criteria, the viewpoint of the supplier and customer, the formulation of a composite equivalence profile, and applications of this profile to problems in vibration testing.

It was shown that current testing practices and test specifications have several known weaknesses, the fundamental ones including the following:

1. High-impedance fixtures are often used in an attempt to obtain test repeatability, but they may lead to grossly conservative results.

2. The same accelerated tests are sometimes used for both life (endurance) and performance demonstration purposes.

3. Time-scale changes used to define an accelerated test are not always based on well-founded procedure.

4. The practice of enveloping field data to provide a test excitation spectrum ignores interaction between equipment and its supporting structure.

5. The use of complicated test procedures when the economics of the situation does not warrant their use.

Equivalence techniques find several uses. Once a test specification has been established, the equivalences provide a means to estimate resulting dynamic design loads in terms of an equivalent stress level. Simpler substitute tests may be derived when the test specification is unnecessarily complex. The effects of test time acceleration may be predicted, and comparisons may be made of the relative fragility of different equipment or design modifications. Under limited circumstances the equivalence techniques also provide a means of predicting equipment performance or structural response to different sets of test requirements.

None of the vibration equivalences have universal applicability. Each must be limited to situations where a single predominant failure mode is most adequately described by the selected equivalence technique. The damaged-based and magnitude equivalences cannot be applied independently but must be used in conjunction with an interaction technique.

The interaction equivalence techniques are as yet only attempts to approximate field conditions, and no general mechanical impedance equivalence has gained acceptance. As a result, current testing practices which employ a simple motion control input to a test specimen are reasonably accurate if the specimen's supporting structure is not influenced by the presence of the specimen. If, however, the apparent weight of the equipment approximates or exceeds that of the supporting structure, severe overtesting can result from motion-control test practices. In this situation the design engineer may decide to attempt a mechanical-impedance simulation. In the first case he may be forced into creating an unnecessarily heavy or costly design. In the second case he has the problem of proving that the selected mechanical-impedance equivalence technique is valid.

In addition to the concepts of force control, equivalent foundation weight, and response limiting, Piersol [131] suggested that multiaxial responses need to be considered. That is, laboratory excitations and responses are usually uniaxial, whereas field excitations and responses are multiaxial. The relationships between uniaxial and multiaxial excitations and responses are not understood and need further study.

Mechanical-impedance equivalence techniques are generally impractical because in-service force data are not available or easy to obtain. There have been some attempts to recognize the impedance problem, for example MIL-STD-810B, Method 514, Paragraph 4.0, will allow input acceleration level reductions based on specimen weight. Even though a reliable approach to determining a general purpose mechanical-impedance equivalence has yet to be developed, there is much interest in doing so [132-136].

Current work in fatigue and wearout phenomena is based primarily on cumulative-fatigue theory. The basic mechanism of fatigue damage is unknown. A poor understanding of the mechanics by which fatigue damage actually accumulates has resulted in several theories that approximate observed failure behavior. As pointed out by Mains [67], it will be possible to accommodate other forms of wearout in the generalized equivalence expressions once wearout models can be defined mathematically.

The fatigue-based equivalences were created by assuming that it is possible to describe a representative environment in a form which may be manipulated in the equivalence relationships. Swanson [137] has suggested that proper representation of actual service history is possible only by analysis of a very large sample of load history. He suggested that because load histories consist of different types of loading, each of which has unique characteristics, that equivalence may exist in one type of loading; however, equivalences between loading types are tenuous. Crandall [138] noted that there is no equivalence between first-passage failures for random and sinusoidal histories. A sinusoid has a first passage, that is, a critical stress or deformation level which is exceeded during the first cycle, or never. He noted also that the random processes have a mean time to first-passage failure with a large dispersion about the mean.

The user of this monograph should recognize several important characteristics of the vibration equivalence techniques: (a) Each type of vibration equivalence is not valid in a general sense but must be applied to the specific situation for which it was derived, (b) if a vibration equivalence is to be used, it must be related to the primary damage process or malfunction expected during the test, (c) a complex specimen may require the application of more than one type of equivalence and the limiting characteristic may shift between critical locations and with test level and response frequency, and (d) it is not reasonable to expect high accuracy from vibration equivalences when in-service data, material properties, and the production uniformity all have significant variability which contribute to dispersion of test data.

A good test was defined as one which fails equipment which will fail in service and will not fail equipment which is satisfactory for service. The definition was found to be deceptively simple. To perform a good test we must be able to describe the environment and be able to duplicate it in a test laboratory. In addition, it is necessary to formulate a suitable framework of specifications so that contractors and subcontractors can legally define project responsibilities and produce hardware to perform an intended function.

Several weaknesses exist in our ability to perform a good test. Most are unavoidable. However, it is necessary to discuss them so that they may be avoided to the largest extent possible. The primary weaknesses result from a lack of adequate field data, the use of motion-control test practices which ignore the interaction between equipment and structure, attempts to write general specifications, and approaches used to accelerate test time.

The concept of accelerated life testing must be used with consideration of the type of failure of malfunction which may result. In those cases where malfunction is level dependent it may be advisable to test equipment performance at a lower test input level than is used to simulate total service endurance. In many instances efforts to design a good test raise many more questions than answers, and it is necessary to live with unpleasantries such as data scatter and empiricism.

This monograph was prepared to present a current overview of related but heretofore scattered activities in an important area of vibration testing. Vibration-equivalence techniques are those approaches which are taken to predict and duplicate the results of service vibration conditions. The review of literature relevant to vibration equivalence represents the thinking of many qualified persons who have turned their attention to various facets of vibration equivalence. It is evident that much progress has been made but more is needed.

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APPENDIX
SYMBOLS AND NOTATION

A	A constant defined in the text
AF	Amplification factor
AG	Acceleration gradient
b	Negative reciprocal slope of logarithmic *S-N* diagram
B	Bandwidth
c	Damping
C	A constant defined in the text
D	Damage
$\bar{D}$	Dynamic modulus
e	Distance from the neutral axis
exp ( )	Natural or naperian e raised to the bracketed power
E	Modulus of elasticity
$E[\ ]$	Expected value of quantity in brackets
f_c	Characteristic frequency
f_n	Natural frequency
F	Force
$\bar{F}$	RMS force
g	The local acceleration of gravity
h	Crack depth
$H(\ )$	Response magnification function
I	Moment of inertia
$\bar{I}$	A stress interaction factor
K	Spring rate
ℓ	Slope of loading spectrum curve, from Lundberg
m	Number of damage nuclei
$\bar{m}$	A generalized mass
M	Mass or moment
$\bar{M}$	Mobility
n	Number of cycles
N	Number of cycles to failure
P	An experimentally determined constant
$P[\ ]$	Probability density of quantity in brackets
q	A scale factor
Q	Resonance transmissibility
r	Rate of crack propagation or ratio of peak sine to rms random stress
R	Ratio of lower to high crack propagation rates
$\bar{R}$	Receptance

s	A scale factor or sweep parameter
S	Stress
$\bar{S}$	RMS stress
t	Time
T	Time to failure
U	Energy
V	A vibration acceleration in g's, i.e., the ratio of the applied acceleration to the local acceleration of gravity
$\bar{V}$	RMS vibration acceleration
W	Work
$\bar{W}$	Apparent weight
$W[\ ]$	The power spectral density of the bracketed quantity
X	Ratio of two quantities
y	A spatial coordinate
$\dot{y}$	Velocity in the spatial coordinate direction
$\ddot{y}$	Acceleration in the spatial coordinate direction
Z	Mechanical impedance
β	Rate of change of compressor speed
η	Number of cycles in a transient
$\Gamma[\ ]$	Gamma function of the type shown in brackets
γ	An experimentally determined factor in Shanley fatigue theory
Δs	Mean stress or change in stress
ϵ	Strain
θ	Impedance angle
λ	Ratio of cycles at higher load to total number of cycles
$\bar{\lambda}$	A constant
σ	Standard deviation
δ	Ratio of actual to critical damping
ω	Angular frequency
ω_n	Natural angular frequency

Superscripts and Subscripts

a	An experimentally determined exponent
d	The Corten-Dolan experimentally determined exponent
e	Equivalent
f	Failure
h	Higher
k	An exponent
ℓ	Level dependent
i, j, m, n	Indexes
o	Original condition
r	Random

s	Stress dependent
sd	Sinusoidal dwell
ss	Sinusoidal sweep
sp	Peak amplitude
x	An experimentally determined exponent in Shanley fatigue theory
α	An experimentally evaluated exponent
γ	An experimental exponent

The terminology and symbolism used to achieve compact notation is as follows:

$$\sum_i V_{sd,i}$$

amplitude of vibration input, g's

index of summation, i.e., there are i different environments

type of vibration, sd = sine dwell, r = random, and ss = sine sweep

$$W[V]_e^{ss}$$

initial history prior to equivalence

condition of equivalence

defines the random environment in terms of power spectral density as a function of frequency for the bracketed parameter

$f_{j,i}$ ⟵ the jth frequency component of the ith vibration history

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54. K. Eldred, W. Roberts, and R. White, "Structural Vibrations in Space Vehicles," WADD Technical Report 61-62, Wright Air Development Division, ARDC, Wright-Patterson AFB, Ohio, Dec. 1961.

A discussion of the vibration response of a space vehicle to various excitation sources. An extensive bibliography on equivalences is included.

55. K. M. Eldred, "Vibroacoustic Environmental Simulation for Aerospace Vehicles," *Shock & Vib. Bull.* 37 (5), 1-11 (Jan. 1968).

Author suggests criteria for determining the minimum size of a portion of a vehicle needed to simulate mounting impedance at components.

56. D. M. Ellett, "Criteria and Standards for Random Vibration," *Shock & Vib. Bull.* 24, 344-347 (Feb. 1957).

Discusses three widely different specifications and questions the Gaussian assumption.

57. J. M. Everitt, R. W. Schock, and J. R. Seat, "Saturn S-11, S-IVB and Instrument Unit Subassembly and Assembly Vibration and Acoustic Evaluation Programs," *Shock & Vib. Bull.* 37 (5), 117-137 (Jan. 1968).

The results of large-scale acoustic and vibration tests are presented. Assembly-type tests are recommended as a technique to improve test accuracy at the component level.

58. T. E. Fitzgerald and L. C. Kula, "Transient Vibration Simulation," *Shock & Vib. Bull.* 37 (4), 59-63 (Jan. 1968).

A sinusoidal sweep has historically been used to simulate long-duration (approx. 1-sec) transient vibration spectra. This was shown to be overly conservative in some cases. A random vibration burst was found to give a better simulation.

59. S. Fogelson, "Digital Analysis of Fatigue Damage to a Multimodal System Subjected to Logarithmically Swept Sinusoidal Vibration Spectra," *Shock & Vib. Bull.* 36 (5), 17-40 (Jan. 1967).

A program for fatigue damage (a Miner damage hypothesis) of a multimodal system is presented by assuming that modal damping is known. No experimental data.

60. J. T. Foley, "Application of Environmental Data to Test Methods–Choice of Test Levels," Sandia Report SC-M-69-287, Albuquerque, New Mexico, July 1969.

A philosophical discussion of the steps involved in arriving at test criteria. The steps are presented in logical manner.

61. W. Forlifer, "Problems in Translating Environmental Data into a Test Specification," *Inst. Environ. Sci. Proc. 1965*, p. 185.

Outlines an approach used in obtaining specifications for launch phase spacecraft testing.

62. P. H. Francis, "The Growth of Surface Microcracks in Fatigue," *ASME Trans., J. Basic Eng.* (Ser. D) 91, 770-779 (Dec. 1969).

Author traces various approaches to fatigue life prediction.

63. P. A. Franken, T. H. Mack, and T. D. Scharton, "Comparison of Mariner Assembly-Level and Spacecraft-Level Vibration Tests," *Shock & Vib. Bull.* 36 (3), 27-38 (Jan. 1967).

Responses of an electronic assembly are presented for two types of vibration tests: (a) mounted to rigid fixture, and (b) mounted in space vehicle. Author suggests the use of gross averages of data for initial comparison of such test.

64. H. O. Fuchs, "A Set of Fatigue Failure Criteria," *ASME Trans., J. Basic Eng.* (Ser. D) 87, 333-343 (1965).

A set of three criteria are developed and presented to be used for the design of long-life parts. Author concludes that (a) crack nucleation, (b) crack propagation, and (c) yielding must be considered for any long-life design.

65. J. C. Furling and H. M. Voss, "Hi Bex Missile Vibration Environment Considerations," *Shock & Vib. Bull.* 35 (7), 25-30 (Apr. 1966).

Authors use acoustic levels at the exterior of a vehicle to predict the external vibration levels. Limited test data are presented.

66. A. E. Galef, "A Quasi-Sinusoidal Vibration Test as a Substitute for Random Vibration Testing," *Shock & Vib. Bull.* 28 (4), 114-119 (Aug. 1960).

Author suggests the use of a modulated sine wave to simulate narrowband random inputs. By running at two different levels, the distribution of peaks is closely approximated.

67. N. Granick, "Status Report on Random Vibration Simulation," *Shock & Vib. Bull.* 27 (2), 137-146 (June 1959).
    The status of random vibration simulation is reviewed critically. The continued use of sinusoidal vibration techniques for the simulation of noise-induced vibration appears justified on the basis of existing knowledge and economic considerations.
68. C. L. Gray, "Feasibility of Using Structural Models for Acoustic Fatigue Studies," *Shock & Vib. Bull.* 30 (4), 140-152 (Apr. 1962).
    The feasibility of employing structural models for acoustic fatigue testing in jet noise environments is examined theoretically and experimentally.
69. C. L. Gray, and A. G. Piersol, "Methods for Applying Measured Data to Vibration and Acoustic Problems," AFFDL-TR-65-60, Air Force Flight Dynamics Laboratory, Wright-Patterson AFB, Ohio, June 1965.
    Specific techniques are outlined for using reduced data to establish coherency estimates, frequency response function estimates, structural vibration predictions, and extreme value predictions.
70. P. Hahn, "Shock and Vibration Considerations in Flight Vehicle System Design," *Inst. Environ. Sci. Proc. 1963*, p. 401.
    Author presents a general discussion of items which must be considered in designing for dynamic environments. Some philosophy relative to types of analyses and tests required is included.
71. R. A. Harmen and J. T. Marshall, "A Proposed Method for Assessing the Severity of the Vibration Environment," *Shock & Vib. Bull.* 26 (2), 259-277 (Dec. 1958).
    A method of analyzing and representing vibration data that defines the environment and its severity to mechanical systems. The proposed method employs three representations: (a) the vibrational intensity of the environment, (b) the susceptibility of any given mechanical system to vibration in general, and (c) a combination of the first two to yield a measure of system response to a particular environment.
72. G. J. Hasslacher and R. C. Kroeger, "The Relationship of Measured Vibration Data to Specification Criteria," *Shock & Vib. Bull.* 31 (2), 49-63 (Mar. 1963).
    A review of the highlights of a successful vibration-data measurement program.
73. G. J. Hasslacher and H. L. Murray, "Determination of an Optimum Vibration Acceptance Test," *Shock & Vib. Bull.* 33 (3), 183-188 (Mar. 1964).
    Describes a method of determining an optimum vibration-acceptance test specification for aerospace electronic equipment wherein items of

equipment are subjected to repeated test sequences at given levels until failure occurs. Data thus produced will yield an acceptance level test which will not fatigue the equipment.

74. P. E. Hawkes, "Response of a Single-Degree-of-Freedom System to Exponential Sweep Rates," *Shock & Vib. Bull.* 33 (2), 296-304 (Feb. 1964).

The peak response and the number of cycles of an SDF system are plotted for several damping conditions. The results are shown as functions of a dimensionless parameter which can be considered to be a measure of the sweep rate of the system.

75. J. A Heinrichs, "Feasibility of Force-Controlled Spacecraft Vibration Testing using Notched Random Test Spectra." *Shock & Vib. Bull.* 33 (4), 47-53 (Mar. 1964).

The author derived test levels for a space vehicle for which the booster-vehicle interface forces were unavailable.

76. R. Heller, "Reliability Through Redundance?" *Inst. Environ. Sci. Proc. 1969*, p. 121.

Reviews work on the reliability of redundant structures including the reliability of redundant structures under fatigue loading.

77. R. W. Hess and H. H. Hubbard, "Acoustic Fatigue Problem of Aircraft and a Discussion of Some Recent Related Laboratory Studies," *Shock & Vib. Bull.* 24, 231-235 (Feb. 1957).

Techniques of random and discrete frequency acoustic testing are discussed and illustrated. Some comparisons of measured stresses and fatigue life are given for a panel configuration exposed to both random and discrete noise.

78. M. H. Hieken, "Vibration Testing of the Mercury Capsule," *Shock & Vib. Bull.* 30 (5), 97-104 (May 1962).

Author describes vibration tests which were conducted on full-scale Mercury capsules. The use of the complete capsule allowed a realistic simulation for internal components.

79. D. E. Hines and D. A. Stewart, "Evaluation of a Design Factor Approach to Space Vehicle Design for Random Vibration Environments," *Shock & Vib. Bull.* 35 (5), 271-306 (Feb. 1966).

Miles analysis was used to obtain experimentally verified design factors for cantilevers subjected to random vibration.

80. G. W. Housner, P. C. Jennings, and N. C. Tsai, "Simulated Earthquake Motions," CIT Report, California Institute of Technology, Pasadena, Calif., Apr. 1968.

Authors present four filtered and time-varying random signals which are used as computational models for earthquakes of various intensities.

81. A. D. Houston, "Internal Vibration of Electronic Equipment Resulting from Acoustic and Shaker-Induced Excitation," *Shock & Vib. Bull.* 37 (3), 7-20 (Jan. 1968).

Responses of several items of electronic equipment were compared under acoustic and mechanical vibration excitation. Responses were found to be higher for the shaker test than for the acoustic test.

82. E. D. Hoyt and G. S. Mustin, "Theoretical and Practical Basis for Specifying a Transportation Vibration Test," *Shock & Vib. Bull.* **30** (3), 122-137 (Feb. 1962).

Authors derive an equivalence using a sinusoidal vibration test for a random environment. A standard vibration test for shipping containers is proposed which is consistent with the theory.

83. N. F. Hunter and J. V. Otts, "Reproduction of Complex and Random Waveforms at Various Points on a Test Item," *Shock & Vib. Bull.* **36** (3), 47-54 (Jan. 1967).

Peak notch filters are used in a manner similar to their use in vibration system equalization to reproduce a desired response at points located on or below a test specimen.

84. N. F. Hunter and J. V. Otts, "Random-Force Vibration Testing," *Shock & Vib. Bull.* **37** (3), 61-74 (Jan. 1968).

Authors present techniques necessary to extend force-controlled tests to random vibration. A passive electrical analog is used to demonstrate the technique.

85. J. R. Hyde and R. A. Schiffer, "Mariner Mars 1964 Acoustically Induced Vibration Environment," *Shock & Vib. Bull.* **35** (7), 31-53 (Apr. 1966).

Authors use acoustic levels to estimate vibration levels on the spacecraft. Test levels were selected at the 95-percentile level and at 4.5 dB above it. Experimental data from flights are well within these envelopes.

86. W. S. Inouye and F. B. Safford, "Vibration Fragility," *Shock & Vib. Bull.* **25** (2), 191-199 (Dec. 1957).

Discusses the application of vibration fragility curves for parts and components by use of superposition to establish an overall fragility limit for electronic equipment.

87. C. Ip, K. Kapur, and B. Slupek, "A Method for Estimating Response of Payload Secondary Structures to Random Excitation," *Inst. Environ. Sci. Proc. 1968*, p. 185.

Authors present a matrix formulation of the spectral density response of secondary structures for correlated loading.

88. H. R. Jaeckel and S. R. Swanson, "Random Load Spectrum Test to Determine Durability of Structural Components of Automotive Vehicles," XII Congress International Des Techniques De L'Automobile, Paper 3-02, Barcelona, España, May 1968.

Several studies have shown that road vibrations are essentially random processes. Authors contend that the fatigue testing of automotive components should be done with random vibration.

89. F. A. Jennings, "Practical Applications of Random-Vibration Testing," *Shock & Vib. Bull.* 24, 348-350 (Feb. 1957).
Author states that sinusoidal testing should be used in development tests and that random testing should be used for qualification tests.

90. L. Kaechele, "Probability and Scatter in Cumulative Fatigue Damage," Rand Corp. Memorandum, RM-3688-PR, Dec. 1963.
Author shows how the probability aspect of fatigue can be handled in a manner similar to static properties.

91. A. H. Kaike and S. R. Swanson, "Load History Effects in Structural Fatigue," *Inst. Environ. Sci. Proc. 1969*, p. 66.
Authors review random fatigue loading techniques and suggest an interesting way to accelerate random tests; i.e., eliminate lower level portions of spectrum until article cracks, at which time the low-level components may become significant.

92. S. Kaplan and A. Soroka, "An Approach for Duplicating Spacecraft Flight-Induced Body Forces in a Laboratory," *Shock & Vib. Bull.* 39 (2), 147-156 (Feb. 1969).
Vibration test levels are tailored to fit design loads at various portions of a spacecraft. This approach contrasts with usual tests where levels depend on launch vehicle only and the dynamics of the spacecraft are ignored.

93. A. L. Karneskey and H. C. Schjelderup, "A Combined Analytical and Experimental Approach to AIF," *Shock & Vib. Bull.* 25 (2), 39-54 (Dec. 1957).
Describes an acoustic survey of the RB-66 and A3D aircraft, including the techniques of instrumentation and presentation of data.

94. J. A. Kasuba, "A Realistic Derivation of a Laboratory Vibration Test to Simulate the Overland Transportation Environment," *Shock & Vib. Bull.* 35 (5), 37-48 (Feb. 1966).
An equivalent vibration test for restrained vehicular cargo is derived using the Palmgren-Miner hypothesis. This test is combined with MIL-STD-810 airborne test levels to obtain a single test.

95. H. Katz and G. R. Waymon, "Utilizing In-Flight Vibration Data to Specify Design and Test Criteria for Equipment Mounted in Jet Aircraft," *Shock & Vib. Bull.* 34 (4), 137-146 (Feb. 1965).
Authors propose using separate performance and endurance vibration tests. The performance test (operating) should simulate flight levels and the endurance test (nonoperating) should simulate the expected aircraft life.

96. J. P. Kearns, "The Application of Analysis Techniques to Laboratory Testing," *Shock & Vib. Bull.* 23, 88-92 (June 1956).
Methods of flight vibration analysis are reviewed along with the problem of vibration transmission on a simple structure. Consideration is given to the problem of attempting to correlate failures as produced

by a simulated multifrequency environment with the failures as produced by sine-wave tests.

97. J. Kelleher, "A Proposed Qualification Test Procedure for System Compatibility during Vibration," *Inst. Environ. Sci. Proc. 1960*, p. 91.
A philosophical discussion of the design of a qualification test.

98. D. C. Kennard, "The Correlation of the Effects of Laboratory Versus Service Environments on Hardware," *Shock & Vib. Bull.* 27 (4), 86-87 (June 1959).
A panel discussion presentation which outlines differences between an ideal test and a practical test. Includes data on 32 service failures which were reproduced in the laboratory.

99. W. Kirk, "Improved Reliability Through Acceptance Vibration Testing," *Inst. Environ. Sci. Proc. 1964*, p. 545.
Outlines a type of acceptance test useful in detecting poor workmanship, and develops a scaling law (which may be incorrect) to obtain equivalent levels. He presents data to show how acceptance testing has improved the quality of test items.

100. G. H. Klein and A. G. Piersoll, "The Development of Vibration Test Specifications for Spacecraft Applications," NASA CR-234, May 1965.
An excellent discussion of the general problem of developing vibration test specifications for flight vehicles. Current procedures are reviewed and a logical implementation of state-of-the-art procedures is suggested to arrive at vibration specifications.

101. R. C. Kroeger and L. Marin, "A Preliminary Investigation of the Equivalence of Acoustics and Mechanical Vibrations," *Shock & Vib. Bull.* 30 (4), 103-113 (Apr. 1962).
Authors describe tests of simple structures in a standing wave tube and measurements of acceleration and strain as functions of pressure level and frequency.

102. M. D. Lamoree and J. E. Wignot, "Some Problem Areas in the Interpretation of Vibration Qualification Tests," *Shock & Vib. Bull.* 33 (3), 203-210 (Mar. 1964).
This paper discusses problems concerned with the interpretation of vibration test specifications which arise in conducting qualification tests and in the interpretation of test failures as related to the probability of failure in service.

103. B. J. Lazan, "Energy Dissipation in Structures, With Particular Reference to Material Damping," *Structural Damping*, papers presented at ASME annual meeting, 1959 (J. E. Ruzicka, ed.), American Society of Mechanical Engineers, New York, 1959.
A collection of papers.

104. N. M. L. Lee and J. P. Salter, "The Response of Packaged Military Stores to Truck Transportation–Real and Simulated," Armament Research and

Development Establishment Memorandum 53/65, Fort Halstead, Kent, Nov. 1965.

A discussion of the merits of machine-specified testing rather than motion-specified testing.

105. R. C. Lewis, "Performance Limitations of Available Equipment for Random-Vibration Testing," *Shock & Vib. Bull.* 24, 353-355 (Feb. 1957).

Sinusoidal vibration systems must be derated for random testing due to high peak-to-rms ratios. Systems designed for random testing will be more nearly optimized.

106. C. E. Lifer, "Design of Space Vehicle Structures for Vibration and Acoustic Environments," *Shock & Vib. Bull.* 33 (4), 201-207 (Mar. 1964).

Author recommends that dynamic analyses be conducted on all vehicles to improve design and testing. Uniform methods among all contractors would allow more meaningful interpretation of end results such as reliability predictions and failure analysis.

107. C. E. Lifer and R. G. Mills, "Prediction and Measurement of Vibration Response of the Pegasus Micrometeoroid Measuring Satellite," *Shock & Vib. Bull.* 34 (2), 27-35 (Dec. 1964).

A dynamic analysis of the Pegasus satellite was performed using a lumped-parameter-system model. Verification of the analysis was carried out during the vibration testing.

108. H. Lipsitt, "Crack Propagation in Cumulative Damage Fatigue Tests," Metallurgy and Ceramics Research Laboratory, Wright-Patterson AFB, Ohio, 1963 (AD 471048).

The paper shows that several of the truisms of fatigue are false.

109. E. Lunney, "The Development of the General Environmental Specifications," *Inst. Environ. Sci. Proc. 1963*, p. 193.

A general discussion of items which should be considered in preparing a general specification and procedure.

110. R. M. Mains, "Introduction to Shock and Vibration Simulation," *Shock & Vib. Bull.* 28 (4), 225-231 (Aug. 1960).

Shock and vibration testing or simulation are essential for demonstrating improvements in design, determining adequacy and acceptability of a design, and controlling quality.

111. R. Mains, "Simulation of Shock and Vibration Environments," *Inst. Environ. Sci. Proc. 1961*, p. 38.

Author states that vibration simulation techniques should be based on damage simulation rather than complete simulation of environment.

112. R. M. Mains, "How to Resolve the Problem of Dynamic Design," *Shock & Vib. Bull.* 24, 324-328 (Feb. 1957).

Author states that dynamic design is dependent on three items: (a) load definition, (b) damage criteria, and (c) response prediction. Neglect of any will lead to problems in design.

113. R. M. Mains, "An Application of Accumulative-Damage Criteria," *Shock & Vib. Bull.* **25** (2), 242-256 (Dec. 1957).

Author obtains limiting stress-level predictions for shock spectra using a normal distribution of shock intensities, and concludes that the technique would work for other distributions.

114. P. Marnell and M. Zaid, "Lifetime Evaluation Procedures for Random Shock and Vibration," *Shock & Vib. Bull.* **35** (3), 125-147 (Jan. 1966).

Authors discuss the damage due to a random vibration exposure as obtained by using the Miner hypothesis. Plastic deformation is used as a failure criterion for shock.

115. S. F. Masri, "Cumulative Damage Caused by Shock Excitation," *Shock & Vib. Bull.* **35** (3), 57-71 (Jan. 1966).

The Shanley hypothesis for fatigue-damage accumulation was shown to fit data from three widely different materials (glass, plastic, and steel). The stress data were obtained by allowing damped free vibration of the test specimens.

116. M. Matrullo and R. C. McCaa, "Flight Level Vibration Testing of a Lifting Body Re-entry Vehicle," *Shock & Vib. Bull.* **36** (3), 113-118 (Jan. 1967).

A technique is presented for conducting an acceleration-controlled, force-limited vibration test.

117. K. J. Metzgar, "A Test Oriented Appraisal of Shock Spectrum Synthesis and Analysis," *Inst. Environ. Sci. Proc. 1967*, p. 69.

Author outlines the use of shock spectra in testing, with particular reference made to shock testing on electrodynamic shakers.

118. J. Milne, "A Successful Vibration Test Program for One-of-a-Kind Satellite," *Inst. Environ. Sci. Proc. 1968*, p. 201.

Author uses a combination of analysis and testing to arrive at final qualification test levels. Notched spectra were used to keep from exceeding design criteria loads.

119. F. Mintz, "Random Shake—An Obnoxious Conglomerate or a Delightful Mixture?" *Shock & Vib. Bull.* **24**, 335-336 (Feb. 1957).

Author poses several questions relative to sinusoidal and random vibration. He concludes that sinusoidal testing must be continued where possible since we have so much sine testing equipment in our laboratories.

120. J. Monroe, "A Problem of Sinusoidal vs Random Vibration," *Inst. Environ. Sci. Proc. 1961*, p. 571.

Author develops a random–sine equivalence which gives good results for electronic equipment.

121. C. T. Morrow, "The Significance of Power Spectra and Probability Distributions in Connection with Vibration," *Shock & Vib. Bull.* **28** (4), 171-176 (Aug. 1960).

The two most fundamental descriptions of random vibration are in terms of power spectra and probability distributions. In most cases the spectrum is the more useful quantity. In the case of fatigue the probability distribution is important.

122. C. T. Morrow, "Why Test with Random Vibration," *Shock & Vib. Bull.* 24, 329-332 (Feb. 1957).

Author states that random testing becomes more appropriate as the complexity of the test item increases. A sinusoidal substitute is readily computed for simple test items such as piece parts and metal specimens.

123. C. T. Morrow, *Shock and Vibration Engineering, Volume 1*, John Wiley and Sons, Inc., New York, N.Y., 1963.

Testing may simulate environment, simulate damage, or control the response dynamics of an equipment. Specification writer should understand which approach or approaches he intends to use in a given specification.

124. R. E. Morse, "The Relationship Between a Logarithmically Swept Excitation and the Build-Up of Steady-State Resonant Response," *Shock & Vib. Bull.* 35 (2), 231-262 (Jan. 1966).

An exact mathematical solution is obtained for an SDF system subjected to a logarithmic sweep.

125. R. Mustain, "The Planning of Aerospace Vibration Tests and Programs," *Inst. Environ. Sci. Proc. 1965*, p. 603.

Author presents a philosophical discussion of testing as applicable to large aerospace vehicles.

126. R. W. Mustain, "Statistical Inferences on Environmental Criteria and Safety Margins," *Shock & Vib. Bull.* 29 (4), 274-298 (June 1961).

The paper presents meaningful statistics on the relationships between failure strength and critical stress.

127. R. W. Mustain, "Dynamics Environments of the S-IV and S-IVB Saturn Vehicles," *Shock & Vib. Bull.* 33 (2), 72-88 (Feb. 1964).

A brief review of techniques used to predict the dynamic environments of the S-IV and S-IVB vehicles is presented, and the environments are summarized.

128. R. W. Mustain, "On the Prediction of Dynamic Environments," *Shock & Vib. Bull.* 28 (4), 20-47 (Aug. 1960).

Estimates of jet engine noise, rocket engine noise, and boundary-layer noise have been computed; actual field data are compared with predictions; and vibration levels created by acoustic inputs are presented.

129. D. Muster, "Correlation of the Effects of Laboratory vs Service Environments on Hardware," *Shock & Vib. Bull.* 27 (4), 88-90 (June 1959).

A prepared presentation for panel discussion which outlines the relative impedances of equipment, vehicle, and test machine which will allow a good damage correlation.

130. J. C. New, "Achieving Satellite Reliability Through Environmental Tests," *Inst. Environ. Sci. Proc. 1963*, p. 561.
A general discussion of the testing of limited-production satellites.

131. R. P. Newman, "Multi-Resonance Response to Sine and Random Vibration," *Inst. Environ. Sci. Proc. 1962*, p. 561.
Author experimentally compares response of multi-degree-of-freedom systems to sine vibration, random vibration, and controlled sine spectrum (periodic) vibration.

132. C. E. Nuckolls and J. V. Otts, "A Progress Report On Force Controlled Vibration Testing," *Shock & Vib. Bull.* **35** (2), 17-130 (Jan. 1966).
A test procedure is suggested where force control is used in conjunction with electronic simulation. This allows the test item to react with its environment in a manner similar to actual usage.

133. M. Oleson, "Application of a Special Test Fixture to Vibration Measurement during Static Firing of Rocket Motors," NRL Memorandum Report 1039, Naval Research Laboratory, Washington, D.C., Apr. 1960.
Author uses a special fixture to improve vibration measurements during the static firing of rocket motors.

134. G. Padgett, "Formulation of Realistic Environmental Test Criteria for Tactical Guided Missiles," *Inst. Environ. Sci. Proc. 1968*, p. 441.
Author points out variables which should be considered in arriving at an environmental specification, and shows how such test criteria might be developed for a guided missile.

135. F. Palmisano, "The Value of Acoustical Testing of Small Electronic Components," *Shock & Vib. Bull.* **30** (4), 114-127 (Apr. 1962).
A comparison between acoustic and mechanical tests on several electronic tube types ranging from a single two-element diode to a multielement twin triode.

136. Panel Discussion, "The Relationship of Specification Requirements to the Real Environment," *Shock & Vib. Bull.* **31** (2), 287-301 (Mar. 1963).
A variety of comments on poor specifications, inability to simulate environment, necessity for sizable safety factors, etc.

137. Panel Discussion, "The Specification Problem," *Shock & Vib. Bull.* **34** (4), 153-163 (Feb. 1965).
Good specifications should be conservative without imposing a large factor of safety. Participants agree that test specifications should be standardized between government agencies.

138. Panel Session, "Prediction of Flight Environment," *Shock & Vib. Bull.* **33** (2), 161-171 (Feb. 1964).
The prediction of environmental levels in flight vehicles is more of an art than a science at the present time. It is therefore necessary to have large factors of ignorance in the test levels to account for lack of confidence in the predictions.

139. Panel Session, "The Use of Environmental Data in Design," *Shock & Vib. Bull.* 33 (4), 219-227 (Mar. 1964).

Environmental design is primarily an art. It requires close liaison between designers, analysts, and test engineers.

140. Panel Session, "The Establishment of Test Levels from Field Data," *Shock & Vib. Bull.* 29 (4), 359-376 (June 1961).

Those concerned with determining the probability that given equipment will fail in service are faced with the problem of translating environmental data into realistic laboratory tests. Simulation is the art of laboratory testing to create a condition that is representative of the actual environmental condition to which equipment will be subjected. In this context, a simulated environment is not necessarily similar to the actual environment but rather has the same damage potential.

141. Panel Session, "Standardization of Vibration Tests," *Shock & Vib. Bull.* 33 (3), 219-229 (Mar. 1964).

Standardization of vibration testing procedures should be limited to small, less complex equipment. Many specifications are overly conservative, but this is a specialized problem and should be treated separately from improving reproducibility of tests.

142. A. V. Parker, "Response of a Vibrating System to Several Types of Time-Varying Frequency Variations," *Shock & Vib. Bull.* 29 (4), 197-217 (June 1961).

Author presents an analysis and discussion of the logarithmic sweep function and the so-called log-log sweep function.

143. A. R. Pelletier, "Problems and Considerations in Combining Sine and Random Vibration in the Environmental Test Laboratory," *Shock & Vib. Bull.* 33 (3), 101-106 (Mar. 1964).

A discussion of four basic conditions which must be considered when sweeping sine and random signals are combined into one vibration test environment.

144. R. W. Peverley, "Acoustically Induced Vibration Testing of Spacecraft Components," *Shock & Vib. Bull.* 36 (3), 39-46 (Jan. 1967).

The author presents a technique in which a vibration test was conducted by acoustically exciting a 180° segment of Apollo space module to obtain levels similar to flight measurements.

145. A. G. Piersol, "Nonparametric Tests for Equivalence of Vibration Data," SAE Paper 748C, Sept. 23-27, 1963.

Author uses nonparametric test data analysis techniques to determine if test data are stationary or if measurements represent the same conditions.

146. R. Plunkett, "Problems of Environmental Testing," *Shock & Vib. Bull.* 25 (2), 67-69 (Dec. 1957).

Environmental testing is mainly a comparison process and cannot be expected to qualify systems and components for field service. The designer, inspector, and contract negotiator must recognize the arbitrary character of this type of test and appreciate the proper role of suitable specification waivers and modifications.

147. J. E. Rice, "Interpretation and Application of Specification Requirements that Simulate Vibration Responses of Equipment Being Shipped by Common Carrier," *Shock & Vib. Bull.* 35 (5), 129-132 (Feb. 1966).
Author contends that vibration levels and frequency ranges for large test items (greater than 1000 lb) are unrealistic and that present test criteria will lead to poor design.

148. W. Reimann and W. Wood, "Some Direct Observations of Cumulative Fatigue Damage in Metals," Institute for the Study of Fatigue and Reliability, Report No. 11, Dept. of Civil Engineering and Engineering Mechanics, Columbia University, N.Y., Oct. 1964.
Mixtures of stress types will cause the damage accumulation law to become more complicated.

149. J. Robbins, "Development of a White-Noise Vibration Test for Electron Tubes," *Shock & Vib. Bull.* 23, 251-256 (June 1956).
Author describes a practical white-noise vibration test method for the evaluation of electron tubes.

150. F. Robinson, "Combined Environment Testing of Shipboard Electronic Equipment and Utilization of Regression Analysis," *Shock & Vib. Bull.* 36 (6), 83-90 (Feb. 1967).
A 450-hr combined environment test was performed on electronic modules, and a multiple regression analysis was performed to relate degradation to environmental factors. Degradation was found to be independent of the vibration.

151. J. L. Rogers, "Vibration Tests, an Estimate of Reliability," *Shock & Vib. Bull.* 33 (3), 189-194 (Mar. 1964).
Author discusses vibration testing as a tool for the evaluation of component reliability.

152. T. P. Rona, "Equivalent Vibration Program from the Fatigue Viewpoint," *Shock & Vib. Bull.* 27 (2), 129-136 (June 1959).
Some, if not most, vibration and shock analysis is done to predict the endurance life of structural components. Elementary examples are given and limitations of this approach pointed out.

153. L. W. Root, "Selection of Vibration Test Levels Using Fatigue Criteria," *Shock & Vib. Bull.* 34 (5), 55-65 (Feb. 1965).
Author reviews equivalence equations for like types of vibration and change of time scale techniques using both Palmgren-Miner (PM) and Corten-Dolan (CD) hypotheses.

154. C. V. Ryden, "Summary of Design Margin Evaluations Conducted at the U.S. Naval Missile Center," *Shock & Vib. Bull.* 33 (4), 209-217 (Mar. 1964).

Incremental levels of environmental stresses are applied to missile systems to induce failures deliberately. Improvements in missile reliability are based on such test programs.

155. J. Salter, "A Vibration Exciter Having Generalized Mobility Characteristics," *J. Environ. Sci.* 9, 18 (Aug. 1966).

Author proposes that a multimodal attachment be added to a vibration shaker to obtain an appropriate envelope of peaks. The test item should be able to react in a realistic manner.

156. J. Salter, "Problem Areas in Dynamic Testing," *Inst. Environ. Sci. Proc. 1963*, p. 49.

Author questions validity of present vibration specifications which are based on envelopes of field data and which do not allow tested equipment to react in a normal manner.

157. J. P. Salter, "Advances in Numerology," *Shock & Vib. Bull.* 37 (3), 1-6 (Jan. 1968).

In the author's view the attempt to use precise, carefully defined numerical values to represent the response of a specimen to a given environment is itself unrealistic. The use of simpler and more empirical techniques is urged, and two simple examples are given.

158. J. Schijve, "The Analysis of Random Load-Time Histories with Relation to Fatigue Tests and Life Calculations," *Fatigue of Aircraft Structures; Proceedings.* Symposium on Fatigue of Aircraft Structures, Paris, 1961 (W. Barrois and E. L. Ripley, eds.), Pergamon Press, New York, 1963.

Author presents seven counting techniques for analyzing load-time histories. Several of the techniques yield comparable results; however, the theory is unable to select the most correct technique.

159. H. C. Schjelderup, "A New Look at Structural Peak Distributions Under Random Vibration," WADC-TR-59-676, Wright Air Development Center, Wright-Patterson AFB, Ohio, Mar. 1961.

Author shows how distributions other than Rayleigh might be used in predicting fatigue lives.

160. C. G. Setterlund and J. A. Skoog, "The Bomarc Flight Vibration and its Development into an Equipment Vibration Specification," *Shock & Vib. Bull.* 24, 45-55 (Feb. 1957).

An interim substitute single-frequency vibration test is described where equivalence is based on reproducing rms responses in equipment.

161. G. Setterlund, "Vibration Test Requirements for Prototype Airborne Equipment Items," Boeing Document D-80330, The Boeing Co., Seattle, Wash., May 11, 1956.

A technique is presented for calculating a sinusoidal substitute for a random vibration excitation.

162. F. Shanley, "Discussion of Methods of Fatigue Analysis," *Fatigue of Aircraft Structures; Proceedings,* WADC-TR-59-507, Aug. 1959.

A mathematical theory of fatigue is presented from which stress analysis and design methods can be developed. Theory is based on earlier papers by author.

163. D. T. Sigley, "Sinusoidal Vibration Testing is at Present Adequate," *Shock & Vib. Bull.* 24, 337 (Feb. 1957).

Author believes that sinusoidal vibration testing is adequate since we have a large amount of experience and large quantities of test equipment for sinusoidal tests but have very little experience or equipment for random testing.

164. A. W. Sinkinson, "Designing Electronic Equipment for the Combined Random and Sinusoidal Vibration Environment," *Shock & Vib. Bull.* 34 (2), 137-144 (Dec. 1964).

A Miles analysis is used to obtain an equivalent sine level for a combined sine-random test. No experimental verification is presented.

165. K. W. Smith, "A Procedure for Translating Measured Vibration Environment into Laboratory Tests," *Shock & Vib. Bull.* 33 (3), 159-177 (Mar. 1964).

A Miles analysis was used to obtain equivalence equations including time-scaling equations. The author proposed handling various levels of damping and various fatigue slopes by statistical techniques.

166. E. Soboleski and J. N. Tait, "Correlation of Damage Potential of Dwell and Cycling Sinusoidal Vibration," *Shock & Vib. Bull.* 33 (3), 113-123 (Mar. 1964).

End-loaded cantilever beams of square copper bus bar were subjected to sinusoidal dwell and sweep excitations to obtain an equivalence comparison.

167. C. V. Stahle, "Some Reliability Considerations in Specification of Vibration Test Requirements for Nonrecoverable Components," *Shock & Vib. Bull.* 34 (4), 147-152 (Feb. 1965).

Considers modification of present vibration tests in order to confirm component reliability. Two models of the environment are considered: (a) vibration level defined uniquely, and (b) vibration level defined statistically. In both cases, test times would have to be extended considerably.

168. W. Stronge, "Forced Aperiodic Vibrations," *Inst. Environ. Sci. Proc. 1966*, p. 193.

Obtains the response of a simple system to a displacement input involving variable frequency.

169. C. R. Tallman, "Evaluation of Vibration Problems, Criteria, and Techniques," *Shock & Vib. Bull.* 24, 105-109 (Feb. 1957).

Reviews the present trends in environmental simulation test methods. An overhaul of current specifications is suggested to ensure that tests produce meaningful information.

170. M. C. Trummel, "Ground Test Simulation of Lift-Off and Transonic Vibration Excitation Mechanisms on the Ranger Spacecraft," *Shock & Vib. Bull.* 35 (2), 74-84 (Jan. 1966).

Vibration levels based on estimated acoustic excitation and measured acoustic acceptances are compared to measured vibration levels. Correlation is good above 600 Hz.

171. E. E. Ungar and K. S. Lee, "Considerations in the Design of Supports for Panels in Sonic Fatigue Tests," AFFDL-TR-67-86, Air Force Flight Dynamics Laboratory, Wright-Patterson AFB, Ohio, Sept. 1967.

Authors offer an approach to comparing the structural response of panel supports under sonic excitation.

172. S. Valluri, "A theory of Acoustic Fatigue," Aeronautical Research Laboratories Report ARL 62-370, California Institute of Technology, Pasadena, Calif., July 1962.

Expressions are derived for the prediction of time to failure caused by acoustic excitation. Simplifying assumptions indicate that probability of failure at any time is directly related to the cumulative probability of the distribution function associated with the rms stress response.

173. B. R. Vernier, "Cumulative Damage in Complex Equipment Due to Vibration," *Shock & Vib. Bull.* 25 (1), 165-173 (Dec. 1957).

A process is discussed in which the amplitudes of the acceleration peaks and their probability distributions are used rather than the rms values of the accelerations.

174. M. Vet, "Dwell-Sweep Correlation," *Inst. Environ. Sci. Proc. 1963*, p. 433.

Cantilever beams of four materials were used to obtain fatigue failures at constant input levels.

175. A. Warren, "The Testing of Equipments Subject to Vibration; Some Basic Considerations," Armament Research and Development Establishment Report (L), ARDE, Fort Halstead, Kent, July 1958.

Author bases vibration test levels on similar responses as predicted by an analog computer method. The amplitude and sweep rate of a sine wave are adjusted to yield similar responses to measured accelerations in vehicles.

176. H. R. Welton, R. Carmichael, W. Harger, and L. L. LeBrun, "Definition and Shipping Vibration Environments," *Shock & Vib. Bull.* 30 (3), 27-35 (Feb. 1962).

Covers the status of vibration requirements in packaging specifications and current Aerospace Industries Association (AIA) Committee action.

177. R. W. White, "Theoretical Study of Acoustic Simulation of In-Flight Environments," *Shock & Vib. Bull.* 37 (5), 55-75 (Jan. 1968).

A modal analysis of a uniform, pinned-end, cylindrical shell was performed to predict response accelerations caused by a modified progressive wave, a reverberant field, and a flight acoustic field.

SUBJECT AND AUTHOR INDEX

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